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Tutorial T1
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Blind Audio Source Separation on Tensor Representation

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Tutorial structure

1. **Introduction**
   1. Separation of audio/speech signals
   2. Live demonstration
2. **ICA and IVA**
   1. ICA: Independent Component Analysis
   2. IVA: Independent Vector Analysis
3. **NMF**
   1. NMF: Nonnegative Matrix Factorization
   2. MNMF: Multichannel NMF
4. **ILRMA**
   1. ILRMA: Independent Low-Rank Matrix Analysis

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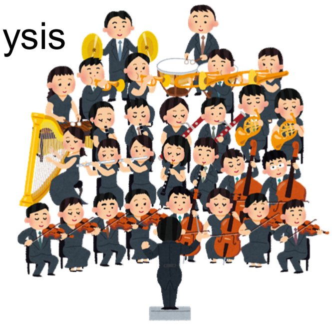
Separation of audio/speech signal

cocktail party effect

speech recognition
in noisy environment

music analysis



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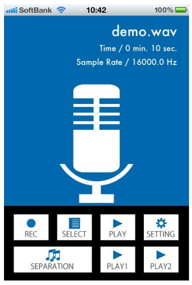
Live demonstration

- Separate 2 speeches with 2 microphones
- iPhone app



- Script

The ICASSP meeting is the world's largest and most comprehensive technical conference focused on signal processing and its applications.

We are demonstrating the Blind Source Separation for convolutive mixtures of speech in a real room with real talkers.

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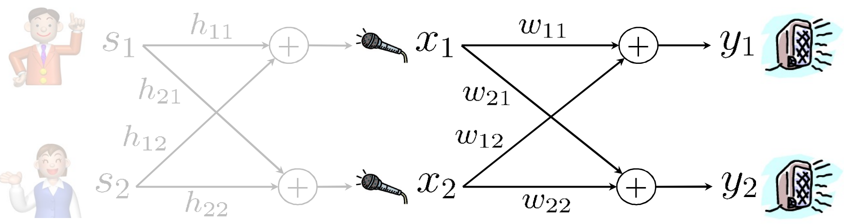
BSS: Blind Source Separation

- Separate the mixtures at microphones x_1, x_2
  - into the original sources s_1, s_2
    - assuming M=N=2 for simplicity
  - source activity and mixing system **H** is unknown (blind)



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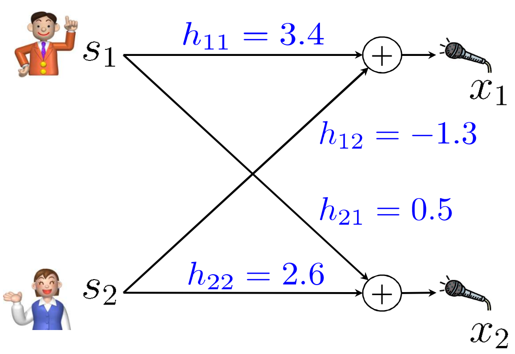
Instantaneous BSS

- Mixing system **H** is described by scalars
- Sources are multiplied by scalars and then mixed

$$x_m(t) = \sum_{n=1}^{N} h_{mn} s_n(t)$$

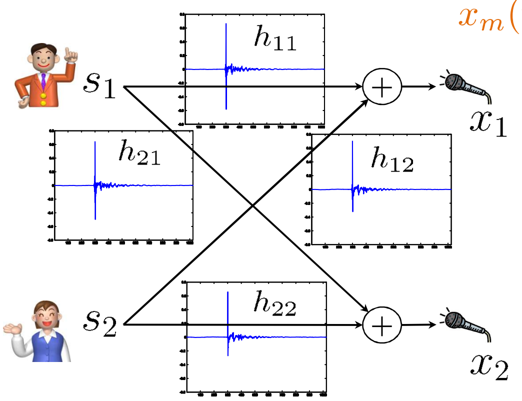


Simple and basic
BSS model

Convolutive BSS

- Delay and reverberations in a real room situation
  - mixing system **H** is described by impulse responses
- Sources are convolutively mixed

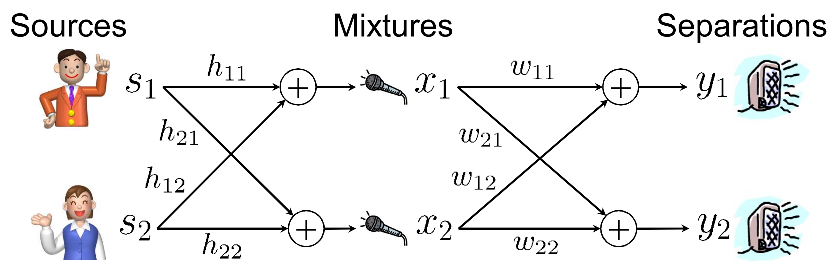
$$x_m(t) = \sum_{n=1}^{N} \sum_{\tau=0}^{L-1} h_{mn}(\tau) s_n(t-\tau)$$



Convolutive BSS is a much harder problem than instantaneous BSS

$$x_m(t) = \sum_{n=1}^{N} h_{mn} s_n(t)$$

The whole system



	Mixing system	**Separation system**
Convolution filter (time domain)	$x_m(t) = \sum_{n=1}^{N} \sum_{\tau=0}^{L-1} h_{mn}(\tau) s_n(t-\tau)$	$y_n(t) = \sum_{m=1}^{M} \sum_{\tau=0}^{L-1} w_{nm}(\tau) x_m(t-\tau)$
Frequency response (frequency domain)	$x_m(i,j) = \sum_{n=1}^{N} h_{mn}(i) s_n(i,j)$	$y_n(i,j) = \sum_{m=1}^{M} w_{nm}(i) x_m(i,j)$

STFT — STFT: short-time Fourier transform

convolution → multiplication

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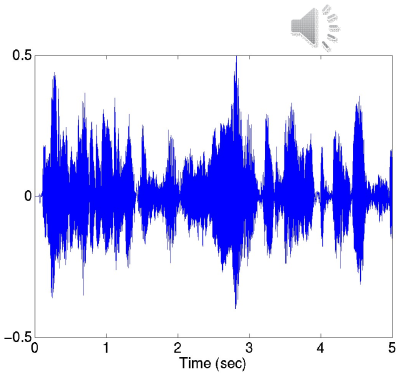
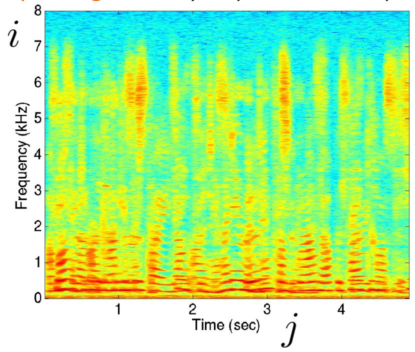
STFT: short-time Fourier transform

- From a time-domain real-valued signal
- To a time-frequency-domain complex-valued signal

$x(t) \in \mathbb{R}$ STFT $x(i, j) \in \mathbb{C}$



Spectrogram: only amplitudes are displayed



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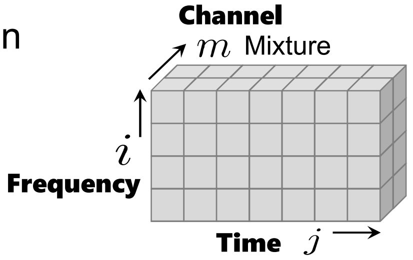
Three key axes

Frequency
- Frequency-domain processing is effective
  - source characteristic
  - convolution → multiplication

Time
- Source activity, Onset and offset

Channel
- Source, Mixture, Separation

Tensor representation



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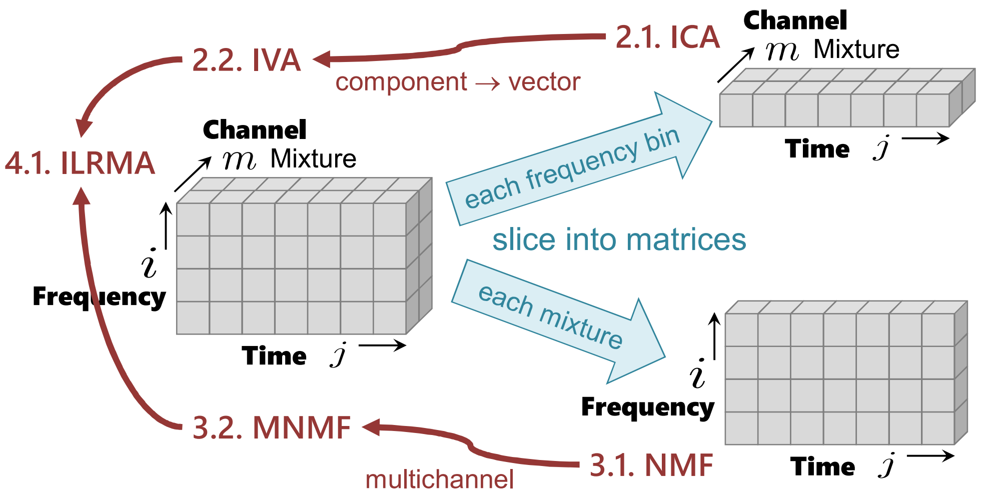
Tensor and sliced matrices



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Notations

s: sources
x: mixtures/observations
y: separations

H: mixing system
W: separation system

i: frequency bin index
j: time frame index
m: microphone index
n: source/separation index

I: number of frequency bins
J: number of time frames
M: number of microphones
N: number of sources

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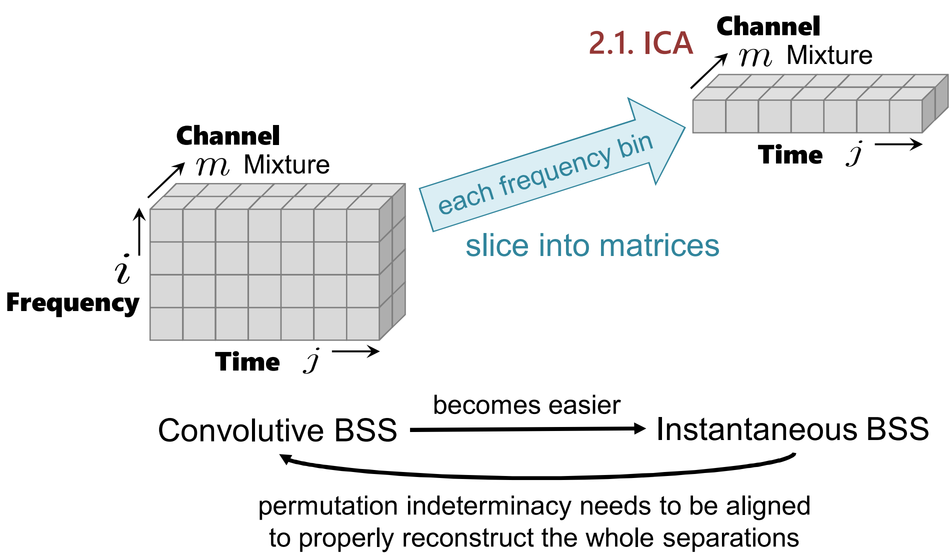
Tensor and sliced matrices



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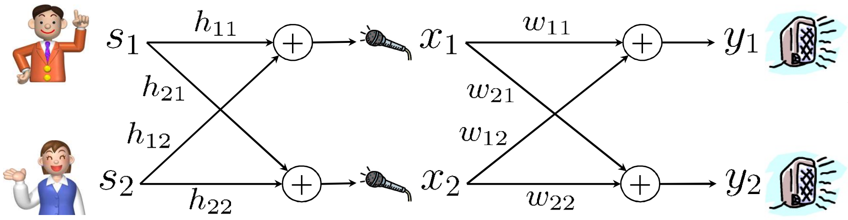
ICA: Independent Component Analysis

- Extract the original sources from the mixtures $x_1,\ x_2$
- Mixing matrix **H** cannot be obtained



Sources are assumed to be independent → Makes y_1, y_2 independent to each other

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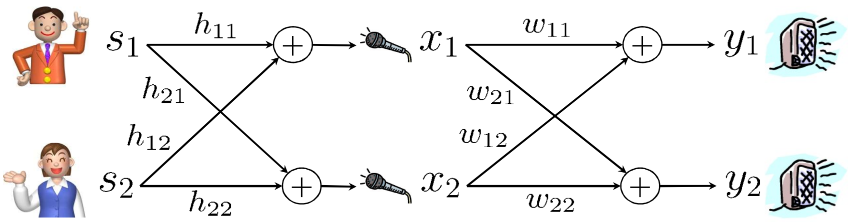
ICA: Independent Component Analysis

- In addition to independence, need to assume
  - source distributions are different from Gaussian



Distributions are different from Gaussian → Makes the distribution far from Gaussian

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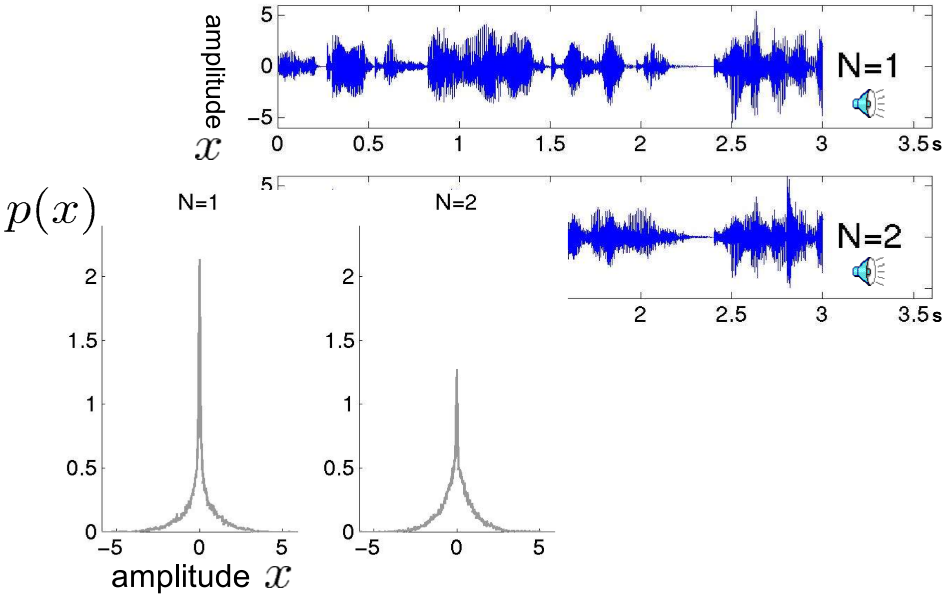
Speech mixtures



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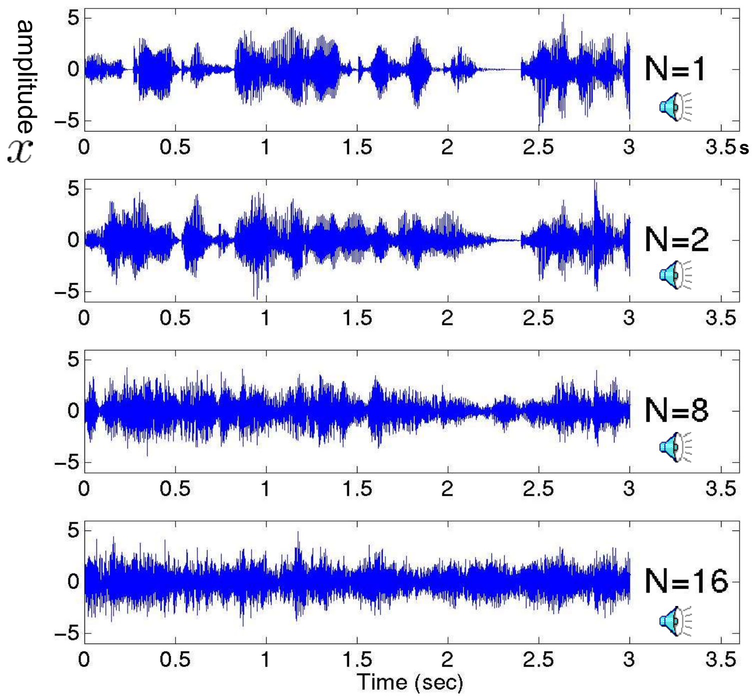

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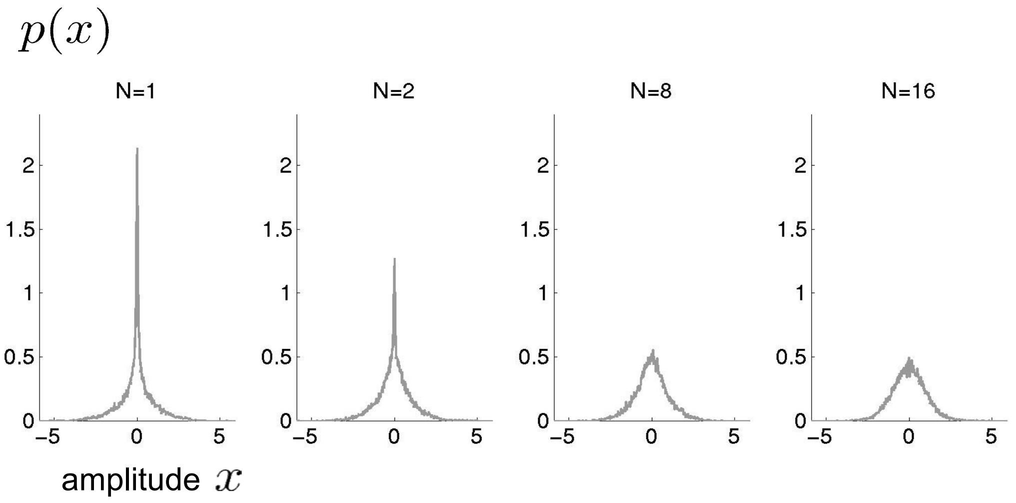
Statistics of the mixtures



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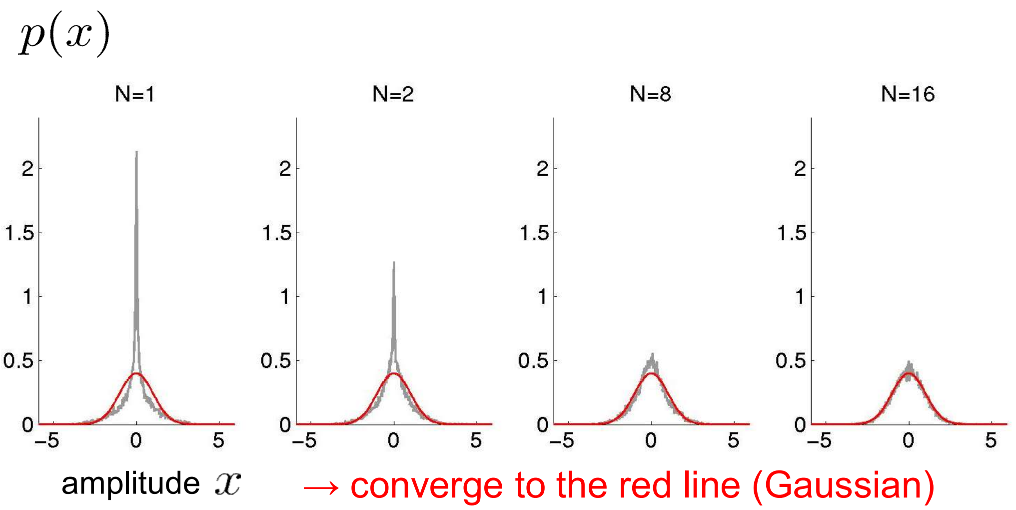
Statistics of the mixtures



→ converge to the red line (Gaussian)

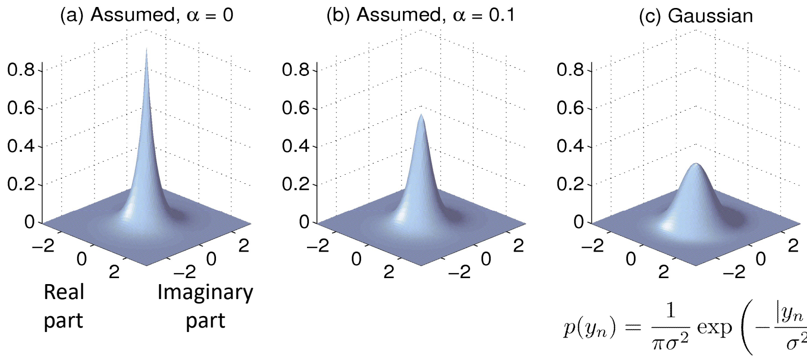
Central Limit Theorem

A complex-valued source model

- Super-Gaussian distribution

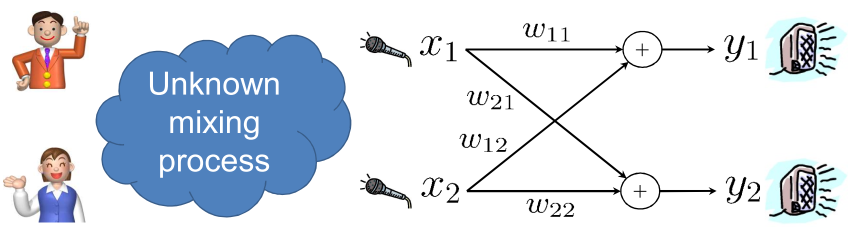
$$p(y_n) \propto \exp\left(-\frac{\sqrt{|y_n|^2 + \alpha}}{b}\right) \qquad y_n \in \mathbb{C}$$

- Sharper peak at the origin than Gaussian



$$p(y_n) = \frac{1}{\pi\sigma^2}\exp\left(-\frac{|y_n|^2}{\sigma^2}\right)$$

Independent component analysis



- Linear operation $\quad \mathbf{y}(j) = \mathbf{W}\mathbf{x}(j)$
- Output independence $\quad p(\mathbf{y}) = \prod_{n=1}^{N} p(y_n)$
- Non-gaussianity $\quad p(y_n) \neq \frac{1}{\pi\sigma^2}\exp\left(-\frac{|y_n|^2}{\sigma^2}\right)$

$$\mathbf{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_M \end{bmatrix} \qquad \mathbf{y} = \begin{bmatrix} y_1 \\ \vdots \\ y_N \end{bmatrix}$$

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Likelihood of separation matrix **W**

- Likelihood of **W** for the whole observations

$$p(\mathcal{X}|\mathbf{W}) = \prod_{j=1}^{J} p(\mathbf{x}(j)|\mathbf{W}) \qquad \mathcal{X} = \{\mathbf{x}(1), \ldots, \mathbf{x}(J)\}$$

- Probability density function, linear transformation

$$p(\mathbf{x}|\mathbf{W}) = |\det \mathbf{W}|^2 \, p(\mathbf{y}) \qquad \leftarrow \quad \mathbf{y}(j) = \mathbf{W}\mathbf{x}(j)$$

- Output independence

$$p(\mathbf{y}) = \prod_{n=1}^{N} p(y_n)$$

Log-likelihood function

$$\log p(\mathcal{X}|\mathbf{W}) = 2J \cdot \log|\det \mathbf{W}| + \sum_{j=1}^{J} \sum_{n=1}^{N} \log p(y_n(j))$$

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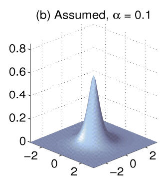
Objective function to be minimized

Maximum likelihood estimation = Minimize the negative log-likelihood

$$\mathcal{J}(\mathbf{W}) = J\left[\sum_{n=1}^{N} \mathrm{E}\{G(y_n)\} - 2\log|\det \mathbf{W}|\right]$$

Contrast function

$$p(y_n) \propto \exp\left(-\frac{\sqrt{|y_n|^2+\alpha}}{b}\right)$$



$$G(y_n) = -\log p(y_n) \qquad G(y_n) = \sqrt{|y_n|^2+\alpha}$$

1st order derivative

$$g(y_n) = \frac{\partial G(y_n)}{\partial y_n^*} = -\frac{\partial \log p(y_n)}{\partial y_n^*} \qquad g(y_n) = \frac{y_n}{2\sqrt{|y_n|^2+\alpha}}$$

2nd order derivative

$$g'(y_n) = \frac{\partial^2 G}{\partial y_n^* \partial y_n} = \frac{\partial g}{\partial y_n} \qquad g'(y_n) = \frac{1}{2\sqrt{|y_n|^2+\alpha}}\left[1 - \frac{1}{2}\frac{|y_n|^2}{|y_n|^2+\alpha}\right]$$

Optimization methods

- Gradient descent

$$\frac{\partial 2\log|\det \mathbf{W}|}{\partial \mathbf{W}^*} = (\mathbf{W}^{\mathsf{H}})^{-1}$$

$$\mathbf{W} \leftarrow \mathbf{W} - \eta \cdot \frac{1}{J}\frac{\partial \mathcal{J}}{\partial \mathbf{W}^*}$$ η :step size

$$\frac{1}{J}\frac{\partial \mathcal{J}}{\partial \mathbf{W}^*} = \mathrm{E}\{\mathbf{g}(\mathbf{y})\mathbf{x}^{\mathsf{H}}\} - (\mathbf{W}^{\mathsf{H}})^{-1}$$

$$\mathbf{g}(\mathbf{y}) = \begin{bmatrix} g(y_1) \\ \vdots \\ g(y_N) \end{bmatrix}$$

  - Expensive matrix inversion
  - Slow convergence

$$g(y_n) = \frac{y_n}{2\sqrt{|y_n|^2 + \alpha}}$$

- Three practical ways
  - **Natural gradient**
  - **Pre-whitening + FastICA**
  - **Auxiliary function-based optimization**

Natural gradient

[Amari et al., 1996]

$$\mathbf{W} \leftarrow \mathbf{W} - \eta \cdot \frac{1}{J}\frac{\partial \mathcal{J}}{\partial \mathbf{W}^*}\mathbf{W}^{\mathsf{H}}\mathbf{W}$$

$$\frac{1}{J}\frac{\partial \mathcal{J}}{\partial \mathbf{W}^*}\mathbf{W}^{\mathsf{H}}\mathbf{W} = \left[\mathrm{E}\{\mathbf{g}(\mathbf{y})\mathbf{y}^{\mathsf{H}}\} - \mathbf{I}\right] \cdot \mathbf{W}$$

$$g(y_n) = \frac{y_n}{2\sqrt{|y_n|^2 + \alpha}}$$

- No matrix inversion
  - Efficient computation
- Equivariance property [Cardoso and Souloumiac, 1996]
  - Free from the characteristics of mixing matrix (e.g. close to singular)

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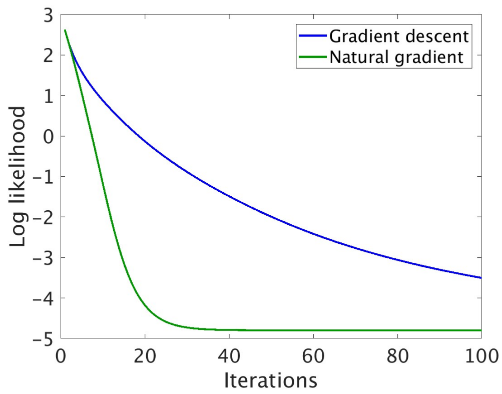
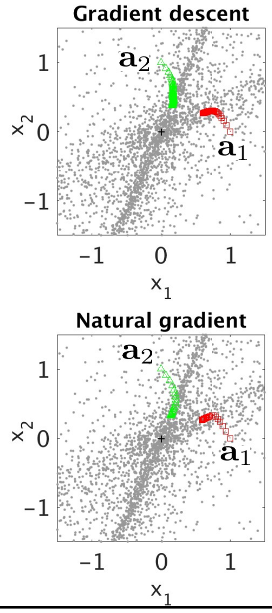
Convergence example

Estimated mixing matrix $\mathbf{A} = \mathbf{W}^{-1}$ ➡

Starting from $\mathbf{W} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Step size: $\eta = 0.1$





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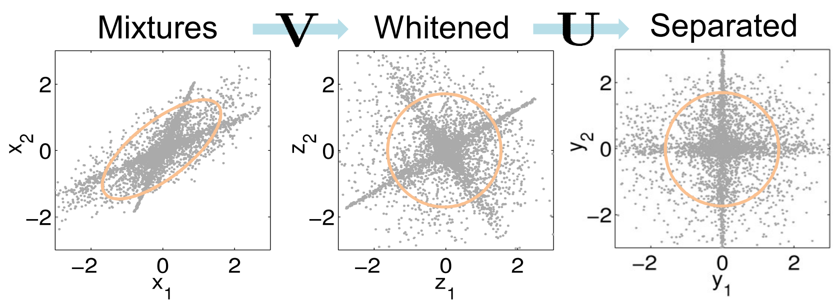
Pre-whitening ＋ FastICA

Separation matrix of the form: $\mathbf{W} = \mathbf{U}\mathbf{V}$ [Hyvarinen et al., 2001]



Pre-whitening

$\mathbf{z}(j) = \mathbf{V}\mathbf{x}(j)$ s. t. $\mathrm{E}\{\mathbf{z}\mathbf{z}^{\mathsf{H}}\} = \mathbf{I}$

via eigenvalue decomposition of $\mathrm{E}\{\mathbf{x}\mathbf{x}^{\mathsf{H}}\}$

FastICA

$\mathbf{y}(j) = \mathbf{U}\mathbf{z}(j)$

Maximize the log-likelihood w.r.t. a unitary matrix **U**

FastICA

- Objective function w.r.t a unitary matrix **U**

$$\mathcal{J}(\mathbf{U}) = J\left[\sum_{n=1}^{N} \mathrm{E}\{G(y_n)\} - 2\log|\det \mathbf{U}|\right]$$
$$= J\sum_{n=1}^{N} \mathrm{E}\{G(y_n)\} \quad \Leftarrow \det\mathbf{U} = 1$$

- Minimize $\mathrm{E}\{G(y_n)\}$ for $y_1, y_2, \cdots, y_N$

$$G(y_n) = \sqrt{|y_n|^2 + \alpha}$$

  - with unitary constraint

$$\mathbf{u}_n^{\mathsf{H}}\mathbf{u}_k = \begin{cases} 1 & \text{if } n = k \\ 0 & \text{if } n \neq k \end{cases} \qquad \mathbf{U} = [\mathbf{u}_1 \cdots \mathbf{u}_N]^{\mathsf{H}}$$

FastICA algorithm

[Hyvarinen et al., 2001]

- For $n = 1, \ldots, N$ (sequentially)
  - Iterate the followings until convergence

Separated signal calculation

$$y_n(j) = \mathbf{u}_n^{\mathsf{H}}\mathbf{z}(j)$$

Optimization of G by Newton's method

$$\mathbf{u}_n \leftarrow \mathrm{E}\{g(y_n)\mathbf{z}\} - \mathrm{E}\{g'(y_n)\}\mathbf{u}_n$$

$$g(y_n) = \frac{y_n}{2\sqrt{|y_n|^2 + \alpha}} \qquad g'(y_n) = \frac{1}{2\sqrt{|y_n|^2 + \alpha}}\left[1 - \frac{1}{2}\frac{|y_n|^2}{|y_n|^2 + \alpha}\right]$$

Gram-schmidt orthogonalization

$$\mathbf{u}_n \leftarrow \mathbf{u}_n - \sum_{k=1}^{n-1}(\mathbf{u}_k^{\mathsf{H}}\mathbf{u}_n)\mathbf{u}_k$$

Unit-norm normalization

$$\mathbf{u}_n \leftarrow \frac{\mathbf{u}_n}{||\mathbf{u}_n||}$$

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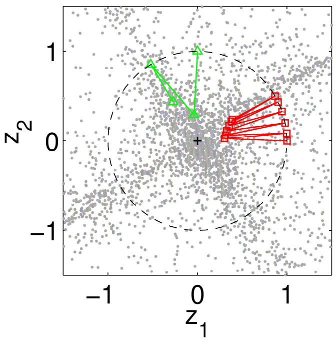
FastICA convergence example

- Red (□)
  - Starting from $\mathbf{u}_1 = [1 \;\; 0]^\mathrm{T}$
  - Likelihood maximization: points close to the origin
  - Unit-norm normalization: points on the unit sphere
  - Good solution only by 5 iterations
- Green (△)
  - Starting from $\mathbf{u}_2 = [0 \;\; 1]^\mathrm{T}$
  - One-step solution by orthogonalization



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Auxiliary function

[Ono and Miyabe, 2010]
[Ono 2011]

- Objective function to be minimized

$$\mathcal{J}(\mathbf{W}) = J\left[\sum_{n=1}^{N} \mathrm{E}\{G(|y_n|)\} - 2\log|\det \mathbf{W}|\right]$$

- If $G(y_n) = G_R(|y_n|)$ and

  $\frac{G'_R(|y_n|)}{|y_n|}$ is monotonically decreasing

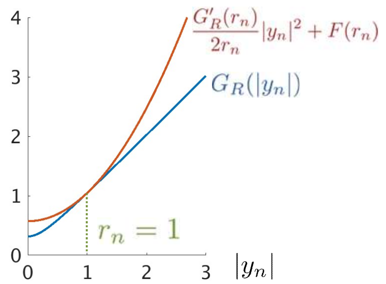
$$G(y_n) = \sqrt{|y_n|^2 + \alpha}$$
$$G_R(|y_n|) = \sqrt{|y_n|^2 + \alpha}$$
$$G'_R(|y_n|) = \frac{\partial G_R}{\partial |y_n|} = \frac{|y_n|}{\sqrt{|y_n|^2 + \alpha}}$$
$$\frac{G'_R(|y_n|)}{|y_n|} = \frac{1}{\sqrt{|y_n|^2 + \alpha}}$$

- Auxiliary function

$$G(y_n) \leq \frac{G'_R(r_n)}{2r_n}|y_n|^2 + F(r_n)$$

  with auxiliary variable r_n

  Equal when $r_n = |y_n|$



Auxiliary function-based optimization

- Iterate the followings until convergence

[Ono and Miyabe, 2010]
[Ono 2011]

Separated signal calculation

$$\mathbf{y} = \mathbf{W}\mathbf{x} \qquad \mathbf{y} = \begin{bmatrix} y_1 \\ \vdots \\ y_N \end{bmatrix} \qquad \mathbf{W} = \begin{bmatrix} \mathbf{w}_1^{\mathsf{H}} \\ \vdots \\ \mathbf{w}_N^{\mathsf{H}} \end{bmatrix}$$

Weighted covariance matrices for each separation

$$\mathbf{V}_n = \mathrm{E}\left\{ \frac{G'_R(|y_n|)}{2|y_n|} \mathbf{x}\mathbf{x}^{\mathsf{H}} \right\} \qquad \frac{G'_R(|y_n|)}{|y_n|} = \frac{1}{\sqrt{|y_n|^2 + \alpha}}$$

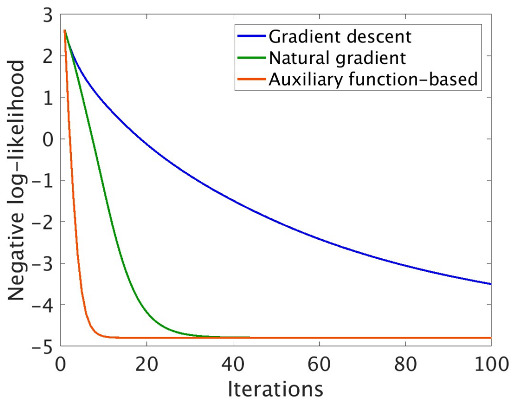
Solve the HEAD problem for $\mathbf{V}_1, \ldots, \mathbf{V}_N$

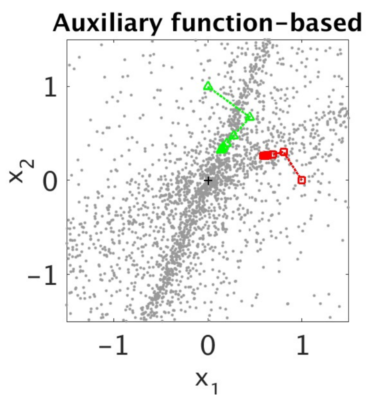
$$\mathbf{w}_m^{\mathsf{H}} \mathbf{V}_n \mathbf{w}_n = \begin{cases} 1 & \text{if } m = n \\ 0 & \text{if } m \neq n \end{cases}$$

Auxiliary ICA convergence example

Estimated mixing matrix $\mathbf{A} = \mathbf{W}^{-1}$

Starting from $\mathbf{W} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$





able to take a big step at an early stage

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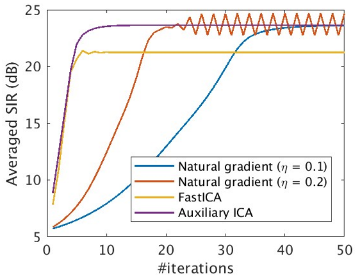
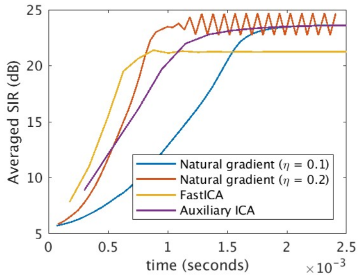
Comparisons of three methods

- Separation of 6-second 3 speeches with 3 microphones
  - measured by signal-to-interference ratio (SIR)
    - at frequency 3586 Hz, 201 samples, pre-whitening applied

convergence behavior



computational cost considered



✓ Natural gradient: sensitive to step-size η

✓ FastICA: fast convergence, slightly limited SIR (unitary constraint)

✓ Auxiliary ICA: fast convergence

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Permutation and Scaling problem

- Ambiguities of ICA solutions

If $\mathbf{y}(j) = \mathbf{W}\,\mathbf{x}(j)$ is a solution, then

$\mathbf{y}(j) \leftarrow \boldsymbol{\Lambda}\,\mathbf{P}\,\mathbf{y}(j)$ is also a solution

for any diagonal $\boldsymbol{\Lambda}$ and permutation $\mathbf{P}$ matrix

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \leftarrow \begin{bmatrix} 3 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

Independence of y_1, y_2, y_3 does not change

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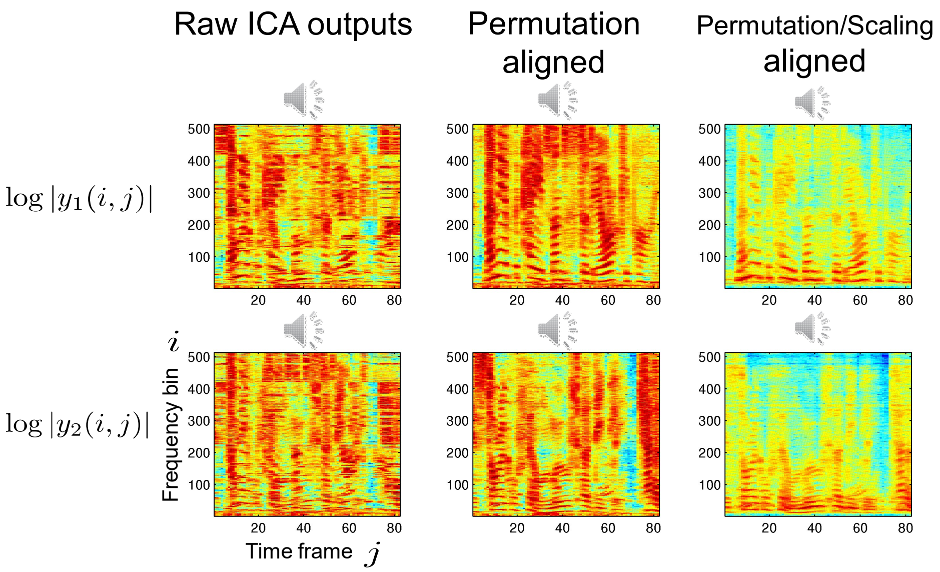
Permutation and Scaling problem



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Solving Permutation and Scaling Problems

- **Permutation**
  - Post-processing
    - see e.g. [Sawada et al., 2004], [Sawada et al., 2011]
  - Tensor methods (IVA, ILRMA)
    - will be explained in later sections
- **Scaling**
  - Refer to a microphone observation
    - So-called "projection back"
    - via mixing system estimation

[Cardoso 1998]
[Murata et al., 2001]
[Matsuoka and Nakashima 2001]
[Takatani et al., 2004]

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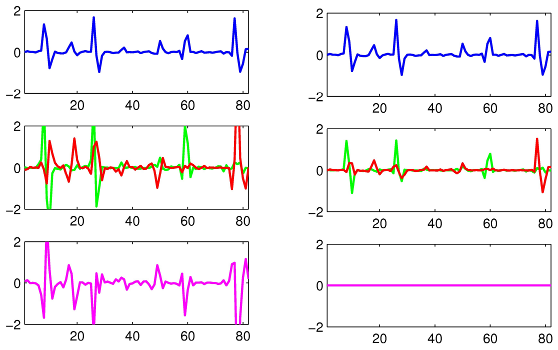
Scaling alignment example

Untouched | **Aligned** $y_n(j) \leftarrow a_{1n} y_n(j)$ | reference microphone $m = 1$

x_1

y_1, y_2

$x_1 - (y_1 + y_2)$



a_{1n} estimated mixing scalar

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Mixing system estimation

Estimated mixing situation

$$\mathbf{x} = \sum_{n=1}^{N} \mathbf{a}_n y_n = \mathbf{A}\mathbf{y}$$

ICA result

$$\mathbf{y} = \mathbf{W}\,\mathbf{x}$$

$$\mathbf{a}_n = \begin{bmatrix} a_{1n} \\ \vdots \\ a_{Mn} \end{bmatrix} \quad \mathbf{A} = [\mathbf{a}_1, \cdots, \mathbf{a}_N] \qquad \mathbf{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_M \end{bmatrix}$$

- How to calculate matrix $\mathbf{A}$
  - If $\mathbf{W}$ has an inverse
    $\mathbf{A} = \mathbf{W}^{-1}$
  - Otherwise ($N < M$)
    $\mathbf{A} = \mathrm{E}\{\mathbf{x}\mathbf{y}^H\}(\mathrm{E}\{\mathbf{y}\mathbf{y}^H\})^{-1}$
    $\mathbf{A} = \mathbf{W}^{+}$

• Least-mean-square estimator that minimizes $\mathrm{E}\{||\mathbf{x} - \mathbf{A}\mathbf{y}||^2\}$

• Moore-Penrose pseudo inverse

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Tutorial structure

1. **Introduction**
   1. Separation of audio/speech signals
   2. Live demonstration
2. **ICA and IVA**
   1. ICA: Independent Component Analysis
   2. IVA: Independent Vector Analysis
3. **NMF**
   1. NMF: Nonnegative Matrix Factorization
   2. MNMF: Multichannel NMF
4. **ILRMA**
   1. ILRMA: Independent Low-Rank Matrix Analysis

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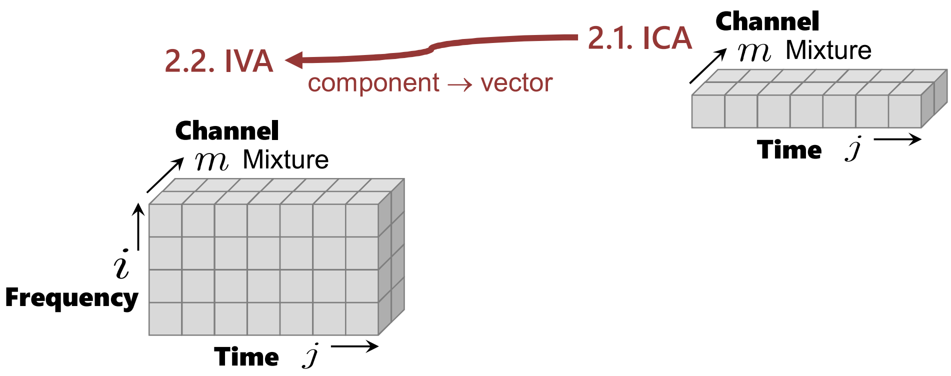
Tensor and sliced matrices



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Two papers at ICA 2006

Independent Vector Analysis: An Extension of ICA to Multivariate Components

Taesu Kim[1,2], Torbjørn Eltoft[3], and Te-Won Lee[1]

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Abstract. In this paper, we solve an ICA problem where both source and observation signals are multivariate, thus, vectorized signals. To derive the algorithm, we define dependence between vectors as Kullback-Leibler divergence between joint probability and the product of marginal probabilities, and propose a vector density model that has a variance dependency within a source vector. The example shows that the algorithm successfully recovers the sources and it does not cause any permutation ambiguities within the sources. Finally, we propose the frequency domain blind source separation (BSS) for convolutive mixtures as an application of IVA, which separates 6 speeches with 6 microphones in a reverberant room environment.

Solution of Permutation Problem in Frequency Domain ICA, Using Multivariate Probability Density Functions

Atsuo Hiroe

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Abstract. Conventional Independent Component Analysis (ICA) in frequency domain inherently causes the permutation problem. To solve the problem fundamentally, we propose a new framework for separation of the whole spectrograms instead of the conventional binwise separation. Under our framework, a measure of independence is calculated from the whole spectrograms, not individual frequency bins. For the calculation, we introduce some multivariate probability density functions (PDFs) which take a spectrum as arguments. To seek the unmixing matrix that makes spectrograms independent, we demonstrate a gradient-based algorithm using multivariate activation functions derived from the PDFs. Through experiments using real sound data, we have confirmed that our framework is effective to generate permutation-free unmixed results.

Concept of "multivariate source model" was presented in two papers at the same conference independently

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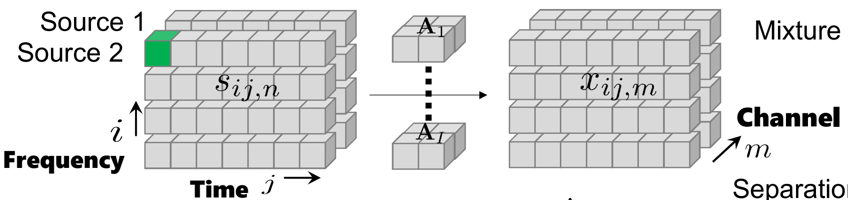
Independent vector analysis

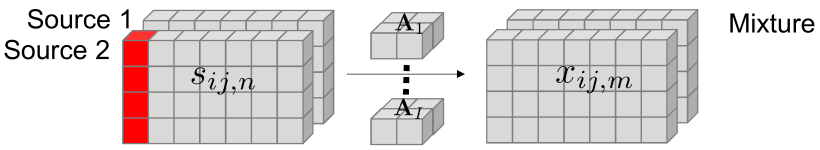
- ICA: Sources generate stochastic scalar variables



Mixing process: $\mathbf{x}_{ij} = \mathbf{A}_i \mathbf{s}_{ij}$

Separation process: $\mathbf{y}_{ij} = \mathbf{W}_i \mathbf{x}_{ij}$

- IVA: Sources generate stochastic vector variables



Same mixing / separation process as ICA, but multivariate source model

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Likelihood of separation matrices **W**

- Likelihood of all **W**s for the whole observations

$$p(\mathcal{X}|\mathcal{W}) = \prod_{j=1}^{J} p(\mathbf{x}_{j,m}|\mathcal{W}) \quad \mathcal{X} = \{\mathbf{x}_{j,m}|j=1,\cdots,J, m=1,\cdots,N\} \quad \mathcal{W} = \{\mathbf{W}_1, \mathbf{W}_2, \cdots, \mathbf{W}_I\}$$

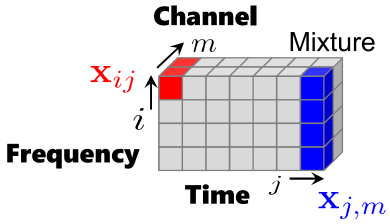
- Probability density function, linear transformation

$$p(\mathbf{x}_{j,1},\cdots,\mathbf{x}_{j,N}|\mathcal{W}) = \prod_{i=1}^{I} |\det \mathbf{W}_i|^2 p(\mathbf{y}_{j,1},\cdots,\mathbf{y}_{j,N})$$

$$\mathbf{y}_{ij} = \mathbf{W}_i \mathbf{x}_{ij}$$

- Output independence

$$p(\mathbf{y}_{j,1},\cdots,\mathbf{y}_{j,N}) = \prod_{n=1}^{N} p(\mathbf{y}_{j,n})$$



Log-likelihood function

$$\mathcal{L} = \log p(\mathcal{X}|\mathbf{W}) = 2J\sum_{i=1}^{I} \log|\det \mathbf{W}_i| + \sum_{j=1}^{J}\sum_{n=1}^{N} \log p(\mathbf{y}_{j,n})$$

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Objective function of IVA

- A set of demixing matrices to be estimated

$$\mathcal{W} = \{\mathbf{W}_1, \mathbf{W}_2, \cdots, \mathbf{W}_I\}$$

- Objective function of IVA

$$\mathcal{J}(\mathcal{W}) = -\mathcal{L}(\mathcal{W}) = \sum_{j=1}^{J}\sum_{n=1}^{N} G(\mathbf{y}_{j,n}) - 2J\sum_{i=1}^{I} \log|\det \mathbf{W}_i|$$

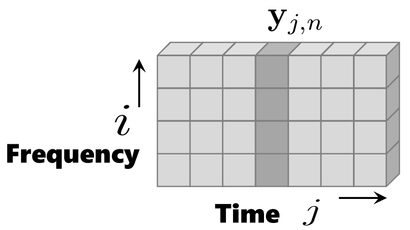
Contrast function

$$G(\mathbf{y}_{j,n}) = -\log p(\mathbf{y}_{j,n})$$

Multivariate p.d.f.

y is a function of w

$$\mathbf{y}_{j,n} = \begin{pmatrix} y_{1j,n} \\ y_{2j,n} \\ \vdots \\ y_{Ij,n} \end{pmatrix} \quad \begin{matrix} y_{1j,n} = \mathbf{w}_{1,n}^{\mathsf{H}} \mathbf{x}_{1j} \\ y_{2j,n} = \mathbf{w}_{2,n}^{\mathsf{H}} \mathbf{x}_{2j} \\ \\ y_{Ij,n} = \mathbf{w}_{I,n}^{\mathsf{H}} \mathbf{x}_{Ij} \end{matrix}$$



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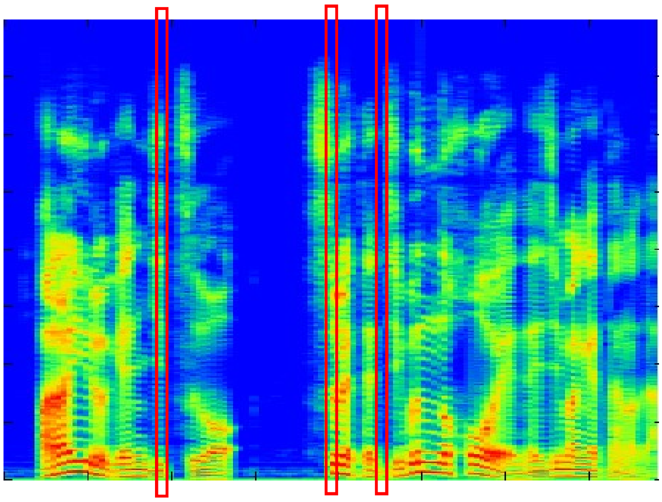
What p.d.f. is appropriate for a spectrum vector?



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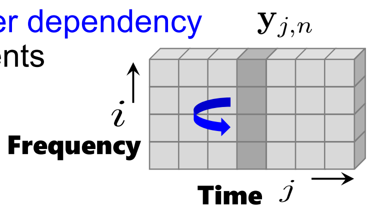
Choice of multivariate p.d.f.

- Necessary properties
  - Non-Gaussian (like ICA)
  - Representing higher-order dependency between vector components



- Well-used multivariate p.d.f.
  - Spherical super-Gaussian [Hiroe 2006],[Kim 2006]

$$p(\mathbf{y}_{j,n}) = C \exp(-||\mathbf{y}_{j,n}||_2)$$

  - Time-varying Gaussian [Ono+ 2012]

$$p(\mathbf{y}_{j,n}) = \frac{C}{\sigma_{j,n}^2} \exp\left(-\frac{||\mathbf{y}_{j,n}||_2^2}{\sigma_{j,n}^2}\right)$$

Variance is time-varying.
Totally it is super-Gaussian.
(Show later)

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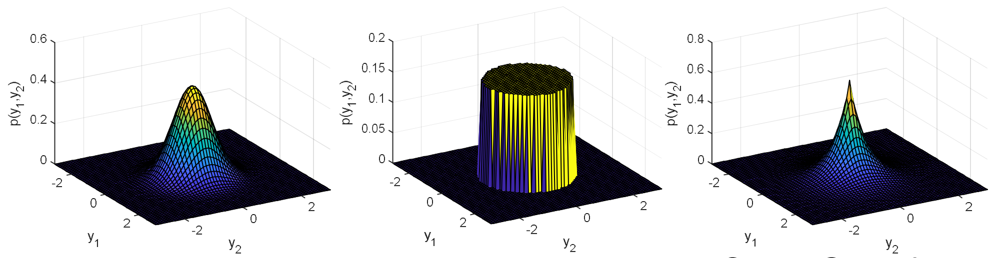
Why spherical super-Gaussian?

- Let $p(\mathbf{y}) = p(y_1, y_2) = q(r)$ where $r = \sqrt{y_1^2 + y_2^2}$
- When $q(r)$ is Gaussian, sub-Gaussian, and super-Gaussian, what dependency between y_1 and y_2 is represented?



Gaussian   Sub-Gaussian   Super-Gaussian

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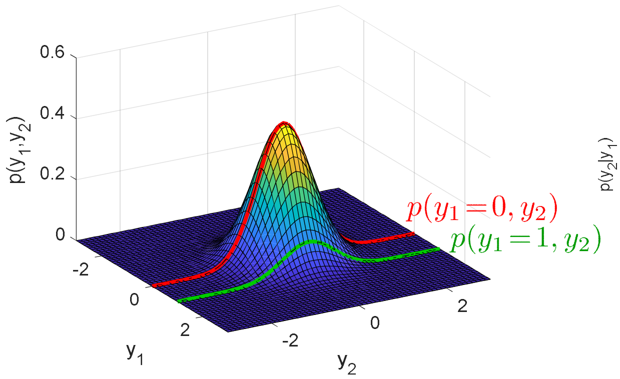
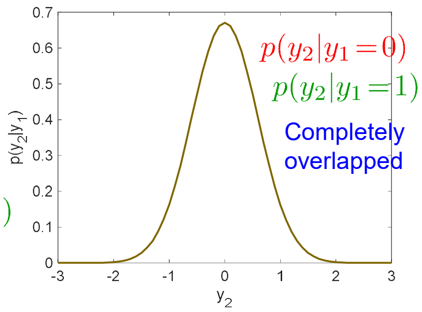
Spherical Gaussian

Joint p.d.f.



Conditional p.d.f.



In spherical Gaussian case, the value of y_1 does not change the p.d.f. of y_2. It means y_1 and y_2 do not have any dependencies.

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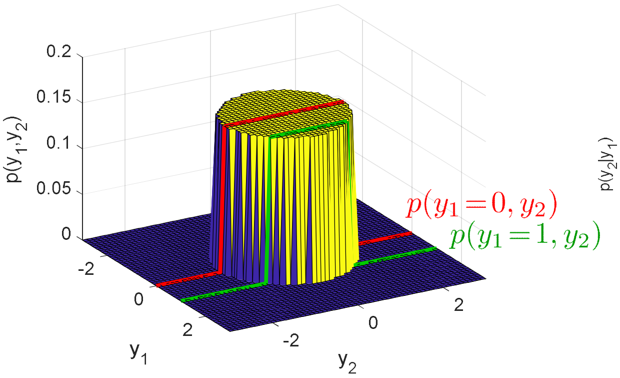
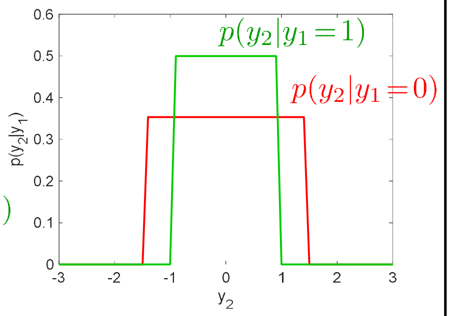
Spherical Sub-Gaussian

Joint p.d.f. | Conditional p.d.f.





In spherical sub-Gaussian case, when $|y_1|$ is larger, $|y_2|$ tends to be smaller oppositely.

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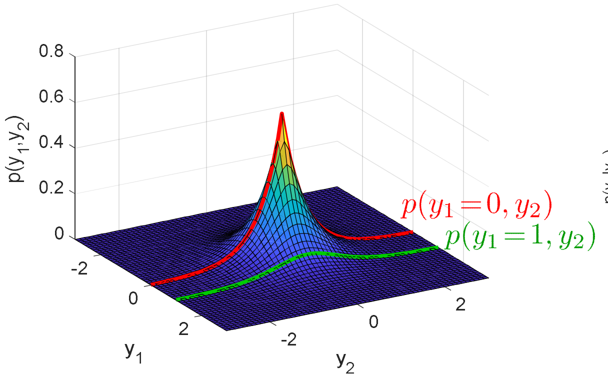
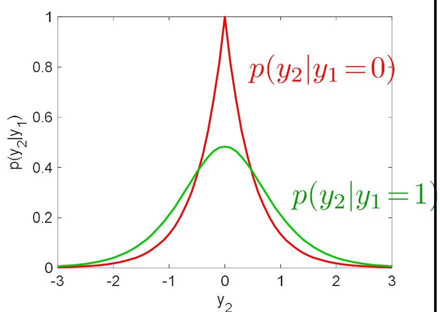
Spherical Super-Gaussian

Joint p.d.f. | Conditional p.d.f.





In spherical super-Gaussian case, when $|y_1|$ is larger, $|y_2|$ tends to be also larger. Therefore, it represents co-occurrence among components.

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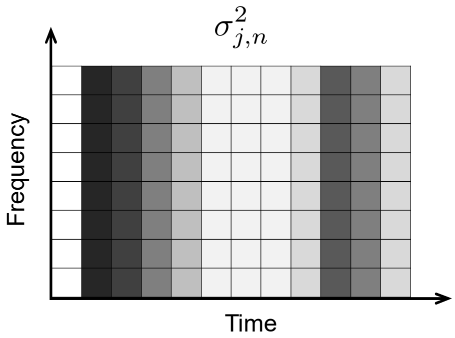
Time-Varying Gaussian

- P.d.f. of time-varying Gaussian

$$p(\mathbf{y}_{j,n}) = \frac{C}{\sigma_{j,n}^2} \exp\left(-\frac{||\mathbf{y}_{j,n}||_2^2}{\sigma_{j,n}^2} \right)$$

- Co-occurrence among frequency components are explicitly represented by $\sigma_{j,n}^2$.
  - Shared among all frequency *i*.
  - Can be changed at each time frame *j*.



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Solutions for IVA

- The objective function of IVA is also nonlinear.

$$\mathcal{J}(\mathcal{W}) = \sum_{j=1}^{J} \sum_{n=1}^{N} G(\mathbf{y}_{j,n}) - 2J \sum_{i=1}^{I} \log |\det \mathbf{W}_i|$$

- Similarly as ICA, three typical methods
  - Natural gradient [Kim+ 2006,Hiroe 2006]
  - Pre-whitening + Fixed-point iteration (FastIVA) [Lee+ 2007]
    - Dr. Taesu Kim (an inventor of IVA)'s code is available https://github.com/teradepth/iva
  - Auxiliary function-based optimization (AuxIVA) [Ono2011, Ono2012b]

Auxiliary Function Approach

- Optimization Problem

  $J(\boldsymbol{\Theta}) \rightarrow \min$

- Auxiliary Function

  $Q(\boldsymbol{\Theta}, \tilde{\boldsymbol{\Theta}}) \geq J(\boldsymbol{\Theta})$

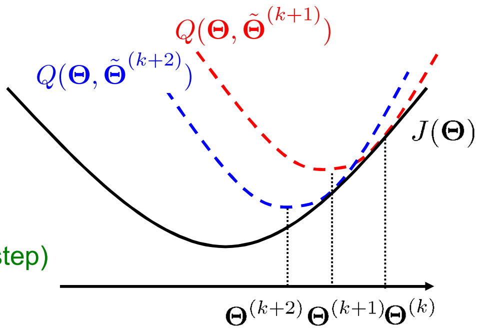
- Alternative Update Rules

  Auxiliary variable update (like E-step)

  $\tilde{\boldsymbol{\Theta}}^{(k+1)} = \mathrm{argmin}_{\tilde{\boldsymbol{\Theta}}}\ Q(\boldsymbol{\Theta}^{(k)}, \tilde{\boldsymbol{\Theta}})$

  Parameter update (like M-step)

  $\boldsymbol{\Theta}^{(k+1)} = \mathrm{argmin}_{\boldsymbol{\Theta}}\ Q(\boldsymbol{\Theta}, \tilde{\boldsymbol{\Theta}}^{(k+1)})$



- Advantages
  - Stable: Convergence is guaranteed
  - Simple: No tuning parameters such as step size

  But how to find useful auxiliary function is problem-dependent

More details will be explained later

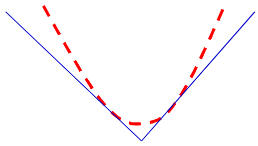
Theorem for quadratic auxiliary function

- If $G(\mathbf{y}) = G_R(||\mathbf{y}||_2)$ and $G'_R(r)/r$ is monotonically decreasing in $r \geq 0$,

  Spherical — Super-Gaussian

$$G(\mathbf{y}) \leq \frac{G'_R(r)}{2r}||\mathbf{y}||_2^2 + F(r)$$

Nonlinear contrast function — Quadratic auxiliary function



holds. The equality sign is valid iff $r = ||\mathbf{y}||_2$.

Examples

Multivariate p.d.f. $p(\mathbf{y})$	Contrast function $G(\mathbf{y}) = -\log p(\mathbf{y})$	Weight function $G'_R(r)/2r$
$Ce^{-\|\|\mathbf{y}\|\|_2^2}$	$\|\|\mathbf{y}\|\|_2^2 = r^2$	1
$Ce^{-\|\|\mathbf{y}\|\|_2}$	$\|\|\mathbf{y}\|\|_2 = r$	$1/2r$
$Ce^{-\log\cosh \|\|\mathbf{y}\|\|_2}$	$\cosh \|\|\mathbf{y}\|\|_2 = \cosh(r)$	$\tanh(r)/2r$

Auxiliary Function for IVA

- Objective Function of IVA

$$\mathcal{J}(\mathcal{W}) = \sum_{j=1}^{J}\sum_{n=1}^{N} G(\mathbf{y}_{j,n}) - 2J\sum_{i=1}^{I}\log|\det \mathbf{W}_i|$$

Nonlinear function of w

If G is spherical and derived from super-Gaussian

- Auxiliary Function for IVA

$$Q(\mathcal{W},\mathbf{r}) = 2J\left[\frac{1}{2}\sum_{i=1}^{I}\sum_{n=1}^{N}\mathbf{w}_{i,n}^{\mathsf{H}}\mathbf{V}_{i,n}\mathbf{w}_{i,n} - \sum_{i=1}^{I}\log|\det \mathbf{W}_i|\right] + F(\mathbf{r})$$

Quadratic function of w

$$\mathbf{r} = \{r_{j,n}\}$$

Source activity (auxiliary variable)

$$r_{j,n} = ||\mathbf{y}_{j,n}||_2 = \sqrt{\sum_{i=1}^{I}|\mathbf{w}_{i,n}^{\mathsf{H}}\mathbf{x}_{ij}|^2}$$

Weighted covariance matrix:

$$\mathbf{V}_{i,n} = \frac{1}{J}\sum_{j=1}^{J}\frac{G'_R(r_{j,n})}{2r_{j,n}}\mathbf{x}_{ij}\mathbf{x}_{ij}^{\mathsf{H}}$$

Minimizing auxiliary function

- The demixing matrix should be updated such that auxiliary function is minimized.

$$Q(\mathcal{W},\mathbf{r}) = 2J\left[\frac{1}{2}\sum_{i=1}^{I}\sum_{n=1}^{N}\mathbf{w}_{i,n}^{\mathsf{H}}\mathbf{V}_{i,n}\mathbf{w}_{i,n} - \sum_{i=1}^{I}\log|\det \mathbf{W}_i|\right] + F(\mathbf{r})$$

$$\frac{\partial Q(\mathcal{W},\mathbf{r})}{\partial \mathbf{w}_{i,n}} = 0 \Rightarrow \mathbf{w}_{i,m}^{\mathsf{H}}\mathbf{V}_{i,n}\mathbf{w}_{i,n} = \delta_{mn} \quad (m = 1,\cdots,N)$$

- From $\frac{\partial Q(\mathcal{W},\mathbf{r})}{\partial \mathbf{w}_{i,n}} = 0$ for all $n = 1,\cdots,N$,

  HEAD (Hybrid Exact-Approximate Joint Diagonalization) problem [Yeredor 2009] is derived.

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HEAD Problem (1/2)

For simplicity, a frequency index *i* is dropped in this slide.

Given N positive definite matrices $\mathbf{V}_1,\ \mathbf{V}_2, \cdots, \mathbf{V}_N$,
find an N × N matrix $\mathbf{W} = (\mathbf{w}_1\ \mathbf{w}_2\ \cdots\ \mathbf{w}_N)^{\mathsf{H}}$ such that

$$\mathbf{w}_m^{\mathsf{H}} \mathbf{V}_n \mathbf{w}_n = \delta_{mn} \quad (m, n = 1, 2, \cdots, N)$$

[Yeredor 2009]

ex. N=3 case

$$\begin{array}{lll} \mathbf{w}_1^{\mathsf{H}} \mathbf{V}_1 \mathbf{w}_1 = 1 & \mathbf{w}_1^{\mathsf{H}} \mathbf{V}_2 \mathbf{w}_2 = 0 & \mathbf{w}_1^{\mathsf{H}} \mathbf{V}_3 \mathbf{w}_3 = 0 \\ \mathbf{w}_2^{\mathsf{H}} \mathbf{V}_1 \mathbf{w}_1 = 0 & \mathbf{w}_2^{\mathsf{H}} \mathbf{V}_2 \mathbf{w}_2 = 1 & \mathbf{w}_2^{\mathsf{H}} \mathbf{V}_3 \mathbf{w}_3 = 0 \\ \mathbf{w}_3^{\mathsf{H}} \mathbf{V}_1 \mathbf{w}_1 = 0 & \mathbf{w}_3^{\mathsf{H}} \mathbf{V}_2 \mathbf{w}_2 = 0 & \mathbf{w}_3^{\mathsf{H}} \mathbf{V}_3 \mathbf{w}_3 = 1 \end{array}$$

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HEAD Problem (2/2)

For simplicity, a frequency index *i* is dropped in this slide.

Given N positive definite matrices $\mathbf{V}_1,\ \mathbf{V}_2, \cdots, \mathbf{V}_N$,
find an N × N matrix $\mathbf{W} = (\mathbf{w}_1\ \mathbf{w}_2\ \cdots\ \mathbf{w}_N)^{\mathsf{H}}$ such that

$$\mathbf{w}_m^{\mathsf{H}} \mathbf{V}_n \mathbf{w}_n = \delta_{mn} \quad (m, n = 1, 2, \cdots, N)$$

[Yeredor 2009]

- Remarks
  - Number of equations = number of variables $= N^2$
  - When $N = 2$, it is equivalent to generalized eigenvalue problem, which can be solved in a closed form. (Show later)
  - When $N \geq 3$, a closed-form solution has never been found.

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Derivation of sequential update rule

For simplicity, a frequency index *i* is dropped in this slide.

Let's consider to solve $\frac{\partial Q(\mathcal{W}, \mathbf{r})}{\partial \mathbf{w}_1} = 0$ in with fixing $\mathbf{w}_2, \mathbf{w}_3$.

$$\mathbf{w}_1^{\mathsf{H}} \mathbf{V}_1 \mathbf{w}_1 = 1$$
$$\mathbf{w}_2^{\mathsf{H}} \mathbf{V}_1 \mathbf{w}_1 = 0$$
$$\mathbf{w}_3^{\mathsf{H}} \mathbf{V}_1 \mathbf{w}_1 = 0$$

Some vector linearly independent of w_2 and w_3

$$\begin{bmatrix} \mathbf{b}^{\mathsf{H}} \\ \mathbf{w}_2^{\mathsf{H}} \\ \mathbf{w}_3^{\mathsf{H}} \end{bmatrix} \mathbf{V}_1 \mathbf{w}_1 = \begin{bmatrix} c \\ 0 \\ 0 \end{bmatrix}$$

$\mathbf{B}$ $\quad c\mathbf{e}_1$

$$\mathbf{B}\mathbf{V}_1 \mathbf{w}_1 = c\mathbf{e}_1$$
$$\leftrightarrow \mathbf{w}_1 = c(\mathbf{B}\mathbf{V}_1)^{-1}\mathbf{e}_1$$

Scale can be fixed by

$$\mathbf{w}_1 \leftarrow \mathbf{w}_1 / \sqrt{\mathbf{w}_1^{\mathsf{H}} V_1 \mathbf{w}_1}$$

If we choose $\mathbf{w}_1$ obtained at the previous iteration as $\mathbf{b}$, $\mathbf{B}$ can be rewritten by $\mathbf{W}$.

$$\mathbf{w}_1 \leftarrow (\mathbf{W}\mathbf{V}_1)^{-1}\mathbf{e}_1$$
$$\mathbf{w}_1 \leftarrow \mathbf{w}_1 / \sqrt{\mathbf{w}_1^{\mathsf{H}} V_1 \mathbf{w}_1}$$

We can update $\mathbf{w}_2, \mathbf{w}_3$ sequentially in the same way.

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Algorithm of AuxIVA

Iterate until convergence [Ono 2011]

For $n = 1 : N$ (every source)

$$y_{ij,n} = \mathbf{w}_{i,n}^{\mathsf{H}} \mathbf{x}_{ij}$$ Update of separation

$$r_{j,n} = ||\mathbf{y}_{j,n}||_2 = \sqrt{\sum_{i=1}^{I} |y_{ij,n}|^2}$$ Update of source activity (shared in all frequency)

For $i = 1 : I$ (every frequency)

$$\mathbf{V}_{i,n} = \frac{1}{J}\sum_{j=1}^{J} \frac{G'_R(r_{j,n})}{2r_{j,n}} \mathbf{x}_{ij}\mathbf{x}_{ij}^{\mathsf{H}}$$ Update of weighted covariance matrix

$$\mathbf{w}_{i,n} \leftarrow (\mathbf{W}_i \mathbf{V}_{i,n})^{-1}\mathbf{e}_n$$
$$\mathbf{w}_{i,n} \leftarrow \mathbf{w}_{i,n} / \sqrt{\mathbf{w}_{i,n}^{\mathsf{H}} \mathbf{V}_{i,n} \mathbf{w}_{i,n}}$$ Update of demixing matrix

Unit vector with the *n*th element unity

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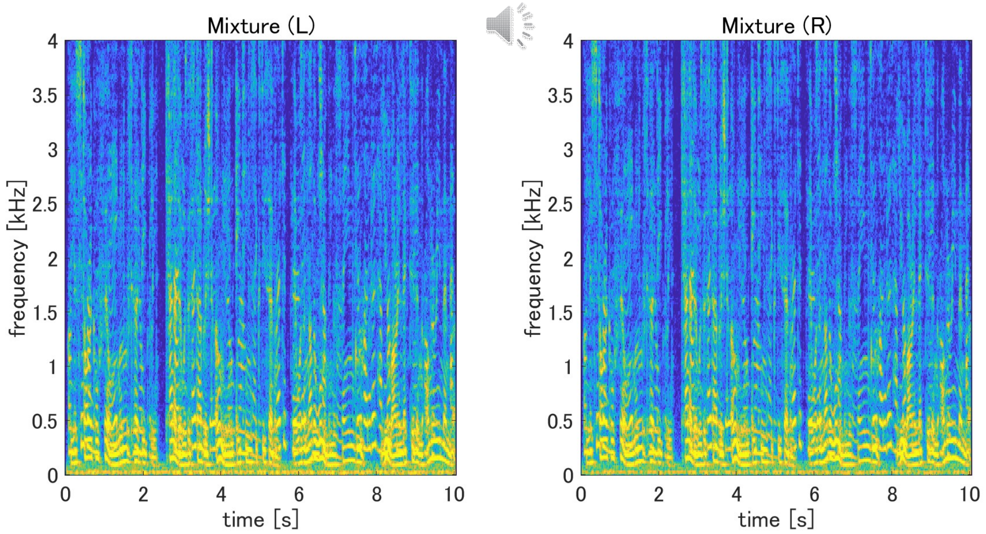
Separation Example (1/6)



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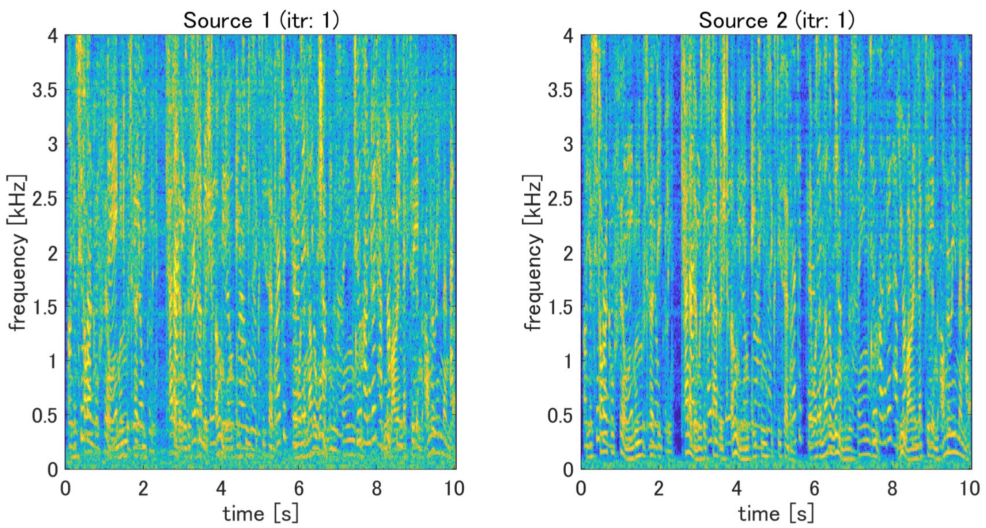
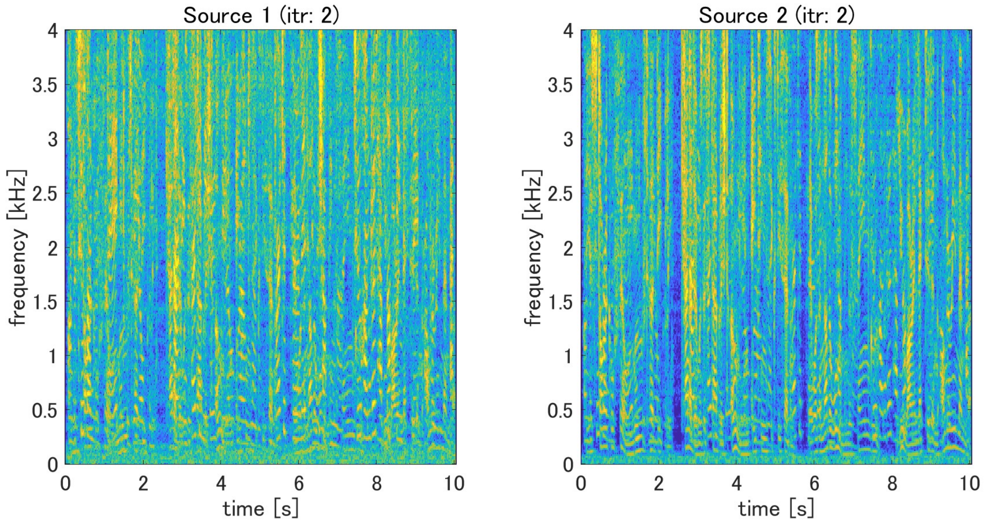
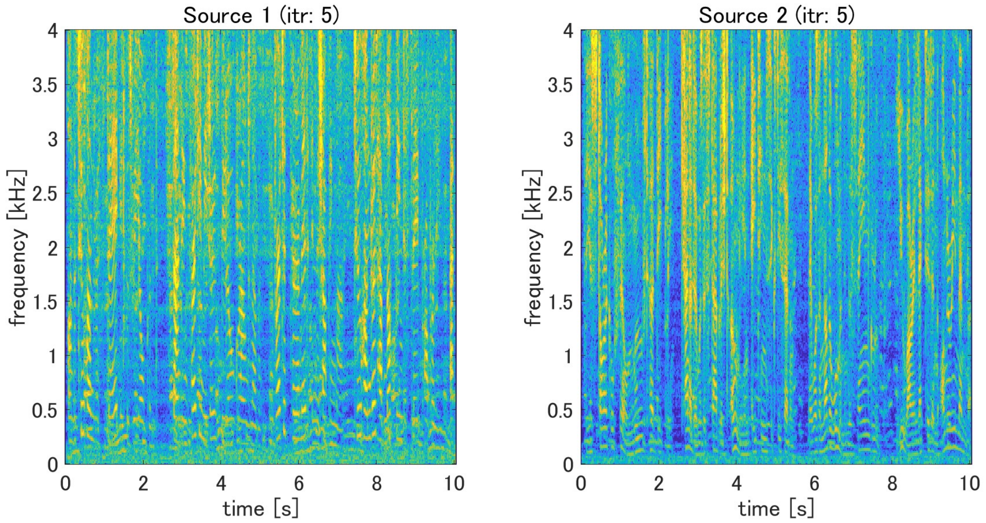
Separation Example (2/6)



Separation Example (3/6)



Separation Example (4/6)



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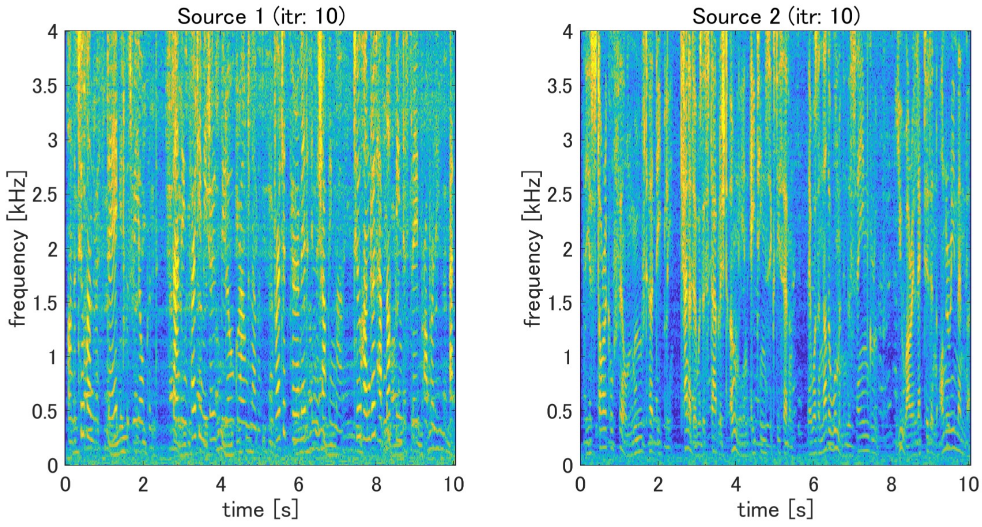
Separation Example (5/6)



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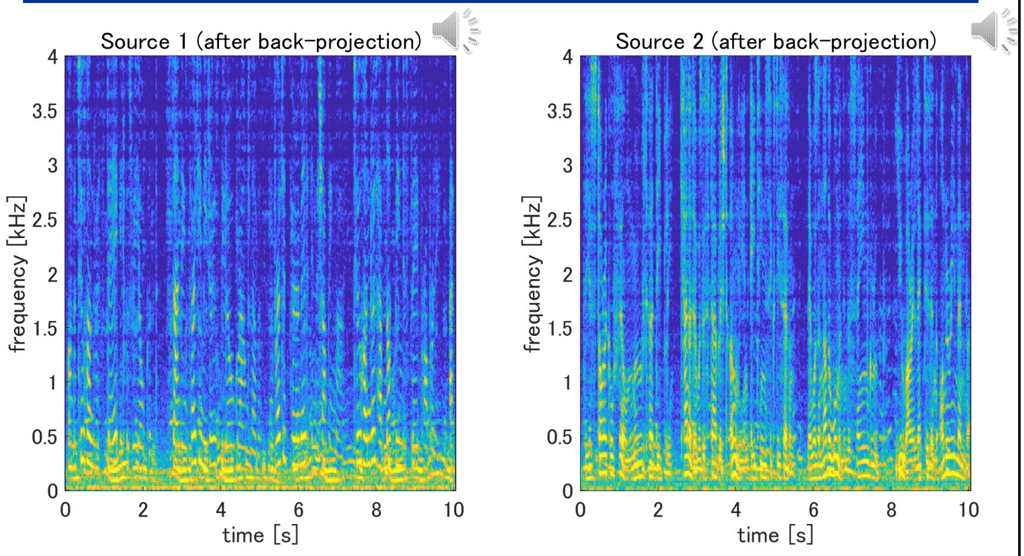
Separation Example (6/6)



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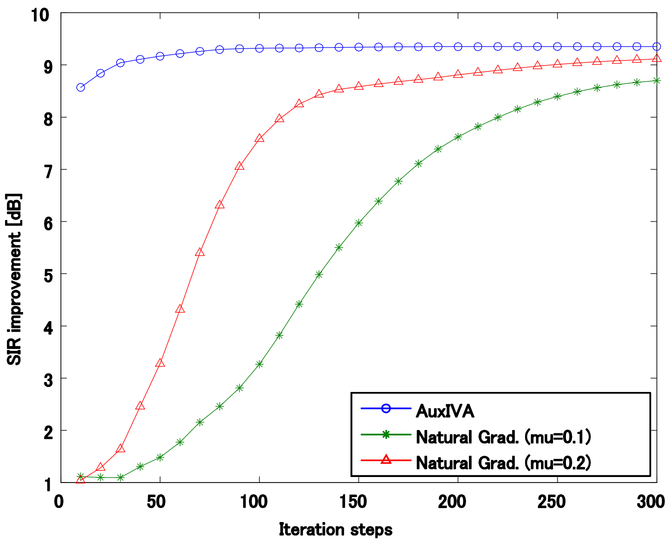
Comparison with Natural Gradient

Three sources case



One iteration (Matlab)

AuxIVA : 0.34s
GradIVA: 0.16s

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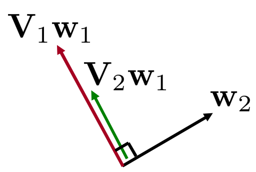
HEAD Problem in Two-Sources Case

$$\mathbf{w}_1^{\mathsf{H}}\mathbf{V}_1\mathbf{w}_1 = 1 \qquad \mathbf{w}_1^{\mathsf{H}}\mathbf{V}_2\mathbf{w}_2 = 0$$
$$\mathbf{w}_2^{\mathsf{H}}\mathbf{V}_1\mathbf{w}_1 = 0 \qquad \mathbf{w}_2^{\mathsf{H}}\mathbf{V}_2\mathbf{w}_2 = 1$$



$$\mathbf{V}_1\mathbf{w}_1 \perp \mathbf{w}_2$$
$$\mathbf{V}_2\mathbf{w}_1 \perp \mathbf{w}_2$$
$$\Rightarrow \mathbf{V}_1\mathbf{w}_1 \parallel \mathbf{V}_2\mathbf{w}_1 \Rightarrow \mathbf{V}_1\mathbf{w}_1 = \lambda\mathbf{V}_2\mathbf{w}_1$$

HEAD problem is deformed to generalized eigenvalue problem, which can be solved in a closed form [Yoshioka 2008,Ono 2010]

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Algorithm of Stereo AuxIVA

Iterate until convergence [Ono 2012b]

$$y_{ij,n} = \mathbf{w}_{i,n}^{\mathsf{H}} \mathbf{x}_{ij}$$

$$r_{j,n} = \|\mathbf{y}_{j,n}\|_2 = \sqrt{\sum_{i=1}^{I} |y_{ij,n}|^2} \quad (n = 1, 2)$$

Update of separation

Update of source activity (shared in all frequency)

$i = 1 : I$

$$\mathbf{V}_{i,n} = \frac{1}{J} \sum_{j=1}^{J} \frac{G'_R(r_{j,n})}{2 r_{j,n}} \mathbf{x}_{ij} \mathbf{x}_{ij}^{\mathsf{H}} \quad (n = 1, 2)$$

Update of weighted covariance matrix

Find $\mathbf{u}_{i,1}, \ \mathbf{u}_{i,2}$ by solving $\mathbf{V}_{i,2} \mathbf{u}_{i,n} = \lambda_{i,n} \mathbf{V}_{i,1} \mathbf{u}_{i,n} \ (\lambda_{i,1} \geq \lambda_{i,2})$

$$\mathbf{w}_{i,n} = \mathbf{u}_{i,n} / \sqrt{\mathbf{u}_{i,k}^{H} \mathbf{V}_{i,n} \mathbf{u}_{i,n}} \quad (n = 1, 2)$$

Update of demixing matrix

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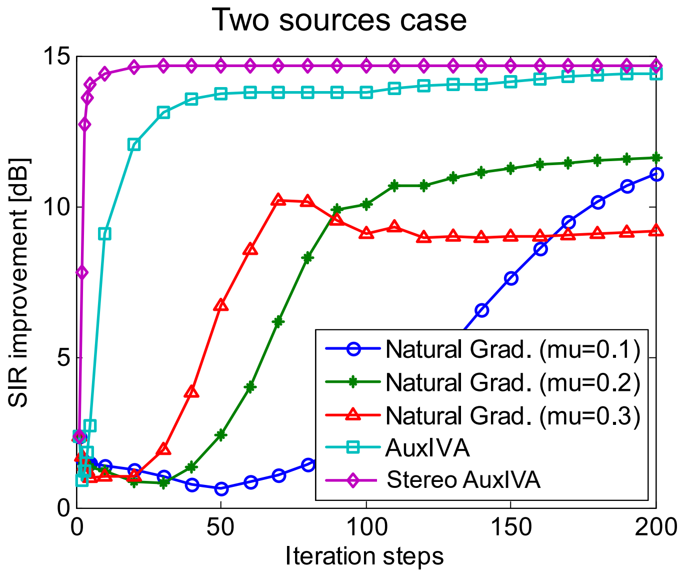
Comparison with Natural Gradient (2)



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Explicit Representation of Eigenvectors of 2 × 2 Matrix

For simplicity, a frequency index *i* is dropped in this slide.

- $\mathbf{V}_2\mathbf{u}_n = \lambda_n \mathbf{V}_1 \mathbf{u}_n \longleftrightarrow \mathbf{H}\mathbf{u}_k = \lambda_n \mathbf{u}_n \ (\mathbf{H} = \mathbf{V}_1^{-1}\mathbf{V}_2)$
- Two eigenvectors of 2 × 2 matrix can be explicitly given by

$$\lambda_1 = \frac{\mathrm{tr}(\mathbf{H}) + \sqrt{\mathrm{tr}(\mathbf{H})^2 - 4\det(\mathbf{H})}}{2}$$

$$\lambda_2 = \frac{\mathrm{tr}(\mathbf{H}) - \sqrt{\mathrm{tr}(\mathbf{H})^2 - 4\det(\mathbf{H})}}{2}$$

$$\mathbf{u}_1 = \begin{pmatrix} H_{22} - \lambda_1 \\ -H_{21} \end{pmatrix}, \quad \mathbf{u}_2 = \begin{pmatrix} -H_{12} \\ H_{11} - \lambda_2 \end{pmatrix}$$

$\lambda_1 \geq \lambda_2$ always holds

- See [Ono 2012b] about fast implementation using vector operation.

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Implementation on iPhone

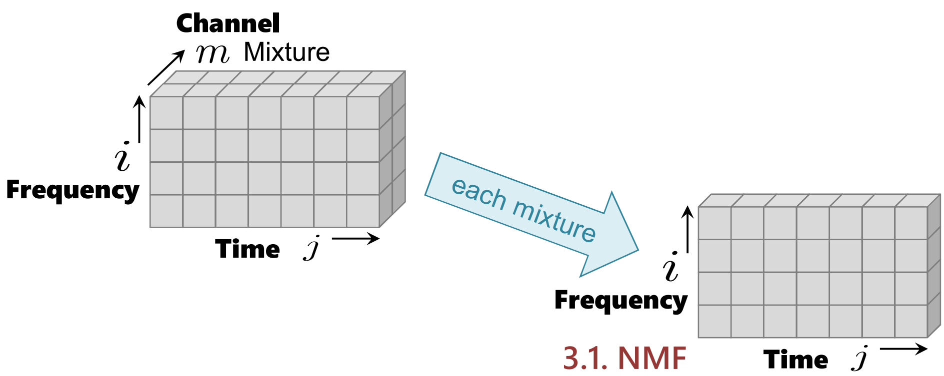
- Stereo AuxIVA was implemented on iPhone [Ono 2012b]
- Calculation time is almost linear to input signal length (RTF≒1/5 @ 16kHz, 10itr. on iPhone4)

- Demo on youtube (https://www.youtube.com/watch?v=ILMbflDMMeE)

Tutorial structure

1. **Introduction**
   1. Separation of audio/speech signals
   2. Live demonstration
2. **ICA and IVA**
   1. ICA: Independent Component Analysis
   2. IVA: Independent Vector Analysis
3. **NMF**
   1. NMF: Nonnegative Matrix Factorization
   2. MNMF: Multichannel NMF
4. **ILRMA**
   1. ILRMA: Independent Low-Rank Matrix Analysis

Tensor and sliced matrices



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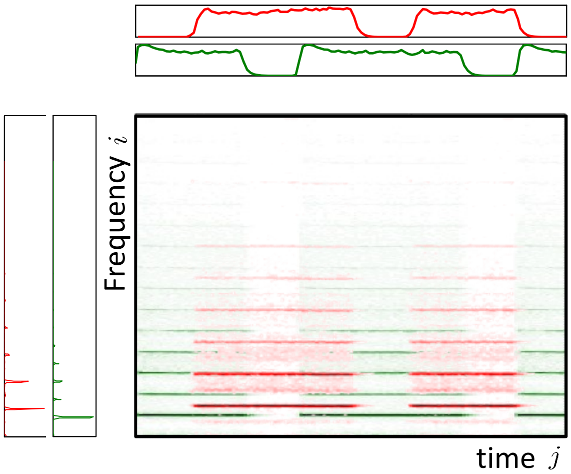
What is NMF? From “generative model” perspective



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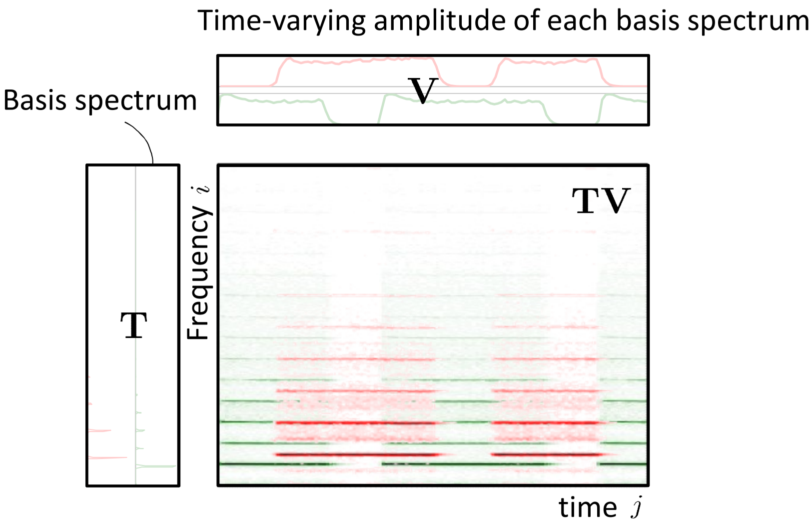
What is NMF? From “generative model” perspective



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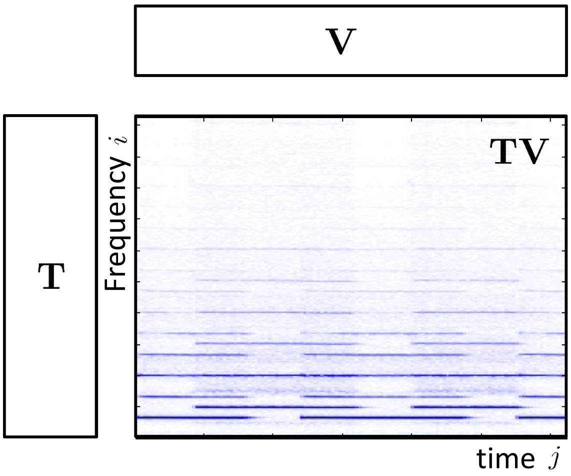
What is NMF? From “generative model” perspective

Source separation = *inverse problem* of estimating $\mathbf{T}$ and $\mathbf{V}$ from $\mathbf{X}$



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NMF as spectrogram model fitting

- Model a mixture spectrum as the sum of basis spectra scaled by time-varying magnitudes

$$x_{ij} \simeq \sum_k t_{jk} v_{kj}$$

Time-varying magnitude

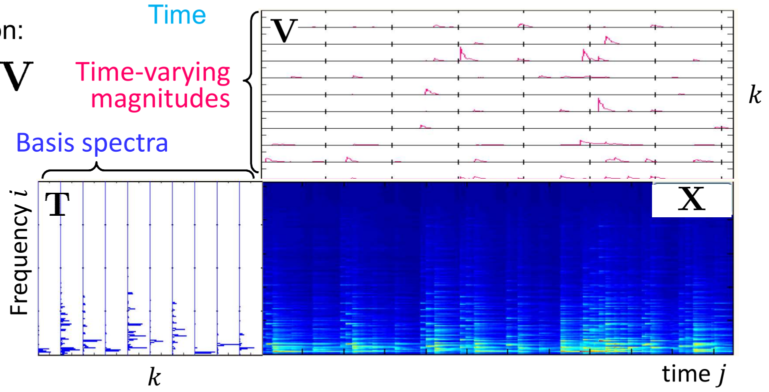
Basis spectrum

Frequency

Time

Matrix notation:

$$\mathbf{X} \simeq \mathbf{TV}$$



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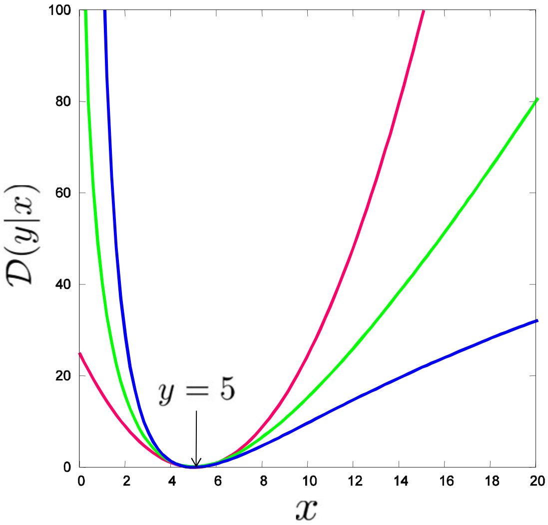
Popular divergence measures

- Measure of difference between x and y



Euclidean distance

$\mathcal{D}(y|x) = (y - x)^2$

Kullback-Leibler (KL) divergence

$\mathcal{D}(y|x) = y \log \frac{y}{x} - (y - x)$

Itakura-Saito (IS) divegence

$\mathcal{D}(y|x) = \frac{y}{x} - \log \frac{y}{x} - 1$

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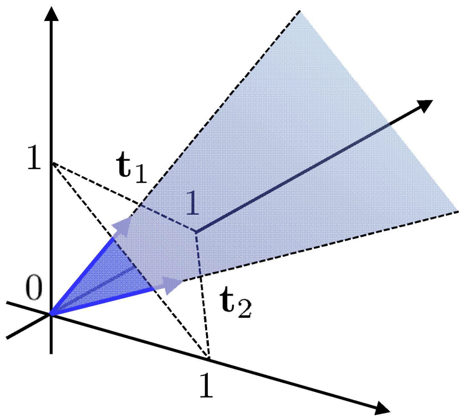
Geometrical understanding of NMF

- Because of the non-negativity of $\mathbf{T}$, all basis vectors lie in the first quadrant.
- Because of the non-negativity of $\mathbf{V}$, $\mathbf{T}\mathbf{v}_j$ can only cover the area (a convex cone) enclosed by the extended lines of all the basis vectors.
- NMF attempts to find a convex cone that is closest to all the observed vectors.



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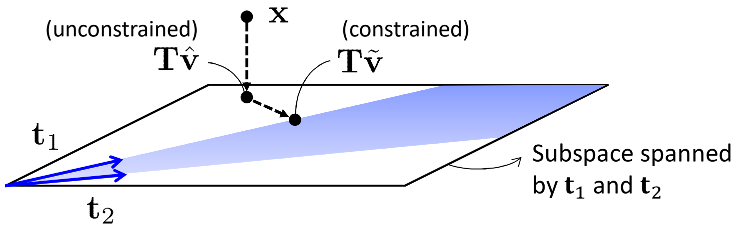
Sparsity-inducing effect of NMF

- NMF naturally produces sparse representations
  - Let $\hat{\mathbf{v}}$ be the solution to an unconstrained optimization problem:
    $$\hat{\mathbf{v}} = \underset{\mathbf{v}}{\operatorname{argmin}}\, \mathcal{D}(\mathbf{x}|\mathbf{T}\mathbf{v})$$
  - $\mathbf{T}\hat{\mathbf{v}}$ corresponds to the closest point from $\mathbf{y}$ in the subspace spanned by $\mathbf{t}_1, \ldots, \mathbf{t}_K$.
  - Except for the case where $\hat{\mathbf{v}}$ is non-negative, the solution to the constrained optimization problem, $\mathbf{T}\tilde{\mathbf{v}}$, will be the closest point to $\mathbf{T}\hat{\mathbf{v}}$ in the area enclosed by the extended lines of all the basis vectors.
  - This means at least one of the elements of $\tilde{\mathbf{v}}$ becomes 0.



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Itakura-Saito divergence NMF [Févotte+2009]

- Model mixture signal as the sum of Gaussian-distributed random signals with rank-1 power spectrograms

Complex spectrogram of component k

$$c_{ijk} \sim \mathcal{N}_{\mathbb{C}}(c_{ijk}|0, t_{ik}v_{kj})$$

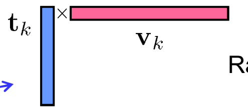
$$x_{ij} = \sum_k c_{ijk}$$

If c_{ijk} and $c_{ijk'}$ are independent

$$x_{ij} \sim \mathcal{N}_{\mathbb{C}}\left(x_{ij}\middle|0, \sum_k t_{ik}v_{kj}\right)$$

→ Likelihood function of $\boldsymbol{T}$ and $\boldsymbol{V}$ $p(\mathbf{X}|\mathbf{T},\mathbf{V})$

Power spectrogram of component k



Rank-1 spectrogram

- Maximum likelihood of $\mathbf{T}$ and $\mathbf{V}$ amounts to NMF using Itakura-Saito divergence

$$\underset{\mathbf{T},\mathbf{V}}{\operatorname{argmax}} \log p(\mathbf{X}|\mathbf{T},\mathbf{V})$$
$$= \underset{\mathbf{T},\mathbf{V}}{\operatorname{argmin}} \sum_{i,j} \left(\frac{|x_{ij}|^2}{\sum_k t_{ik}v_{kj}} - \log \frac{|x_{ij}|^2}{\sum_k t_{ik}v_{kj}} - 1 \right)$$

Itakura-Saito divergence

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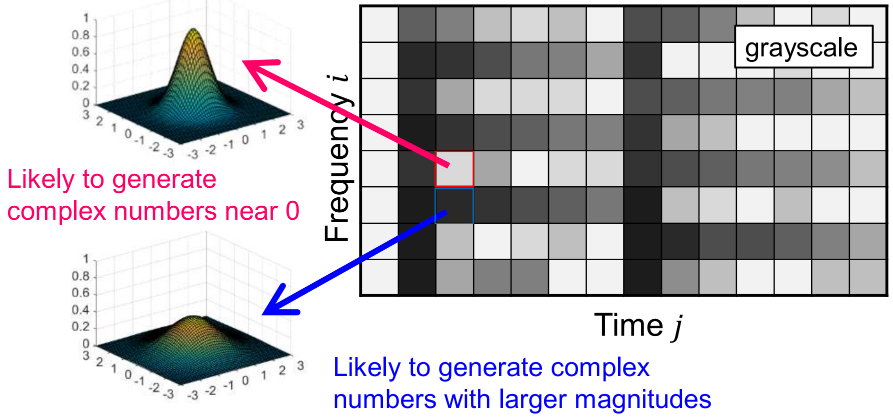
Local Gaussian model (LGM)

- Generative model assuming each element of a complex spectrogram to independently follow a zero-mean complex Gaussian distribution with a different variance (power)

$$c_{ijk} \sim \mathcal{N}_{\mathbb{C}}(c_{ijk}|0, \sigma_{ijk}^2)$$ variance



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Itakura-Saito divergence NMF (IS-NMF)

- Optimization problem:
  Minimize

$$\mathcal{D}_{\mathrm{IS}}(\mathbf{T}, \mathbf{V}) = \sum_{i,j} \left(\frac{|x_{ij}|^2}{\sum_k t_{ik} v_{kj}} - \log \frac{|x_{ij}|^2}{\sum_k t_{ik} v_{kj}} - 1 \right)$$
$$= \sum_{i,j} \left(\frac{|x_{ij}|^2}{\sum_k t_{ik} v_{kj}} + \log \sum_k t_{ik} v_{kj} - \cdots \right)$$

  subject to $\forall i, j, k, \ t_{ik} \geq 0, \ v_{kj} \geq 0$
- How can we solve this?
  - EM algorithm [Févotte+2009]
  - Auxiliary function approach
    - –Majorization-Maximization (MM) algorithm [Kameoka+2006], [Nakano+2010], [Févotte+2011]
    - –Majorization-Equalization (ME) algorithm [Févotte+2011]

Expectation-Maximization (EM) algorithm

- When a certain likelihood function can be written as $p(\mathbf{x}|\theta) = \int p(\mathbf{x}, \mathbf{c}|\theta)\mathrm{d}\mathbf{c}$ where $\mathbf{c}$ is a set of latent variables, a stationary point of $\log p(\mathbf{x}|\boldsymbol{\theta})$ can be found by iteratively performing the following steps:
  - Expectation-step $q(\mathbf{c}|\mathbf{x}) \leftarrow \dfrac{p(\mathbf{x}, \mathbf{c}|\theta)}{\int p(\mathbf{x}, \mathbf{c}'|\theta)\mathrm{d}\mathbf{c}'} = p(\mathbf{c}|\mathbf{x}, \theta)$
  - Maximization-step $\theta \leftarrow \underset{\theta}{\operatorname{argmax}} \underbrace{\int q(\mathbf{c}|\mathbf{x}) \log p(\mathbf{x}, \mathbf{c}|\theta)\mathrm{d}\mathbf{c}}$

$$Q(\theta, \theta') = \mathbb{E}_{\mathbf{c}\sim p(\mathbf{c}|\mathbf{x},\theta')}[\log p(\mathbf{x}, \mathbf{c}|\theta)]$$

Q function

IS-NMF optimization with EM algorithm

[Févotte+2009]

- Likelihood function for IS-NMF:

$$p(\mathbf{x}|\theta) = \int p(\mathbf{x}, \mathbf{c}|\theta)\mathrm{d}\mathbf{c} = \int \underline{p(\mathbf{x}|\mathbf{c})}\,\underline{p(\mathbf{c}|\theta)}\mathrm{d}\mathbf{c} \quad \text{where } \theta = \{\mathbf{T}, \mathbf{V}\}$$

$p(\mathbf{x}|\mathbf{c}) = \prod_{i,j} \delta\left(x_{ij} - \sum_k c_{ijk}\right)$ (independent of θ)

$p(\mathbf{c}|\theta) = \prod_k \prod_{i,j} \mathcal{N}_{\mathbb{C}}(c_{ijk}|0, t_{ik}v_{kj})$

- Q function

$$Q(\theta, \theta') = \sum_k \sum_{i,j} \mathbb{E}_{c_{ijk}\sim p(c_{ijk}|x_{ij},\theta')}\left[\underbrace{\log \mathcal{N}_{\mathbb{C}}(c_{ijk}|0, t_{ik}v_{kj})}_{-\log t_{ik}v_{kj} - \frac{|c_{ijk}|^2}{t_{ik}v_{kj}}}\right]$$

- E-step

$$r_{ijk} \leftarrow \mathbb{E}_{c_{ijk}\sim p(c_{ijk}|x_{ij},\theta')}[|c_{ijk}|^2]$$
$$= t_{ik}v_{kj} - \frac{t_{ik}^2 v_{kj}^2}{\phi_{ij}} + \frac{t_{ik}^2 v_{kj}^2 |x_{ij}|^2}{\phi_{ij}^2} \quad \text{where } \phi_{ij} = \sum_k t_{ik}v_{kj}$$

- M-step

$$\theta \leftarrow \underset{\theta}{\operatorname{argmax}} \sum_k \sum_{i,j} \left(-\log t_{ik}v_{kj} - \frac{r_{ikj}}{t_{ik}v_{kj}} \right)$$

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Auxiliary function approach

- Techniques to find a stationary point of an objective function $D(\theta)$ using an auxiliary function $G(\theta, \alpha)$ that satisfies

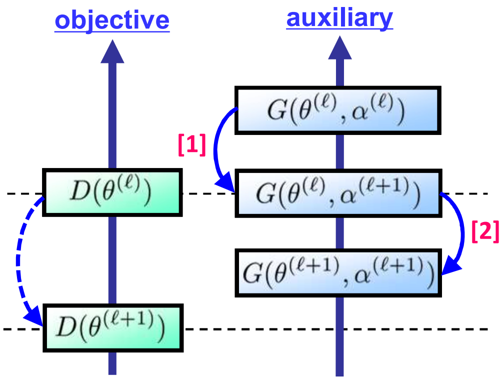
$$D(\theta) = \min_{\alpha} G(\theta, \alpha)$$

- Majorization-minimization
[Hunter & Lange 2004]

[1] $\alpha^{(\ell+1)} = \underset{\bar{\theta}}{\operatorname{argmin}}\, G(\theta^{(\ell)}, \alpha)$

[2] $\theta^{(\ell+1)} = \underset{\theta}{\operatorname{argmin}}\, G(\theta, \alpha^{(\ell+1)})$

$$\begin{aligned} D(\theta^{(\ell)}) &= G(\theta^{(\ell)}, \alpha^{(\ell+1)}) \\ &\geq G(\theta^{(\ell+1)}, \alpha^{(\ell+1)}) \\ &\geq D(\theta^{(\ell+1)}) \end{aligned}$$



- Majorization-equalization
[Févotte & Idier 2010]

[1] $\alpha^{(\ell+1)} = \underset{\bar{\theta}}{\operatorname{argmin}}\, G(\theta^{(\ell)}, \alpha)$

[2] $\theta^{(\ell+1)} \leftarrow \theta \text{ such that } G(\theta, \alpha^{(\ell+1)}) = G(\theta^{(\ell)}, \alpha^{(\ell+1)}),\ \theta \neq \theta^{(\ell)}$

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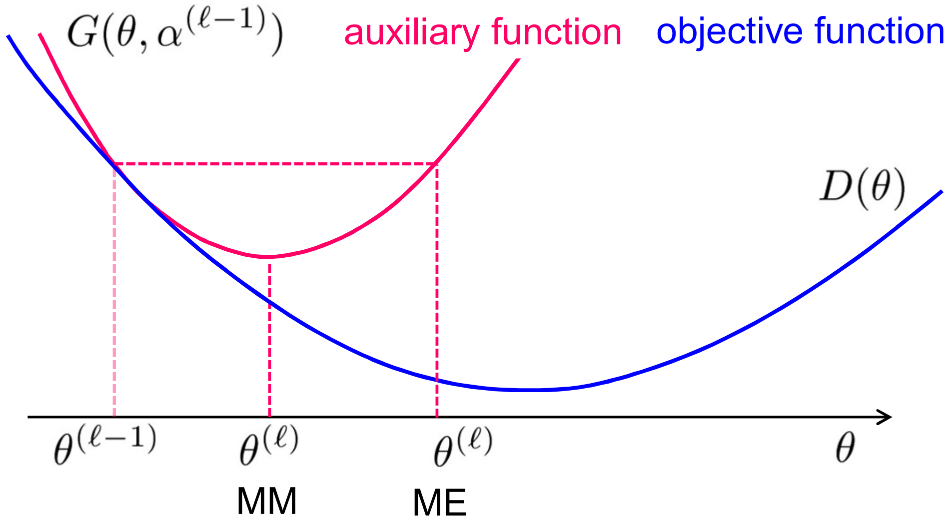
Graphical illustration of MM and ME algorithms

- Comparison between MM and ME updates



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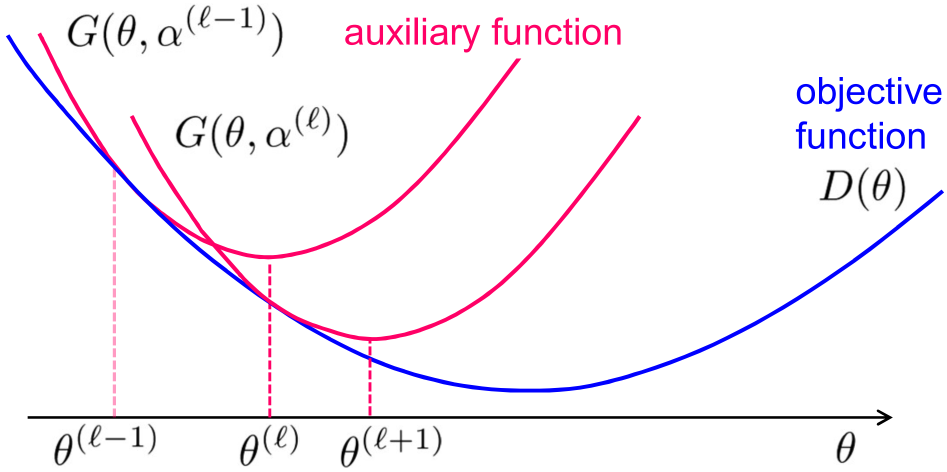
Graphical illustration of MM and ME algorithms

- MM algorithm



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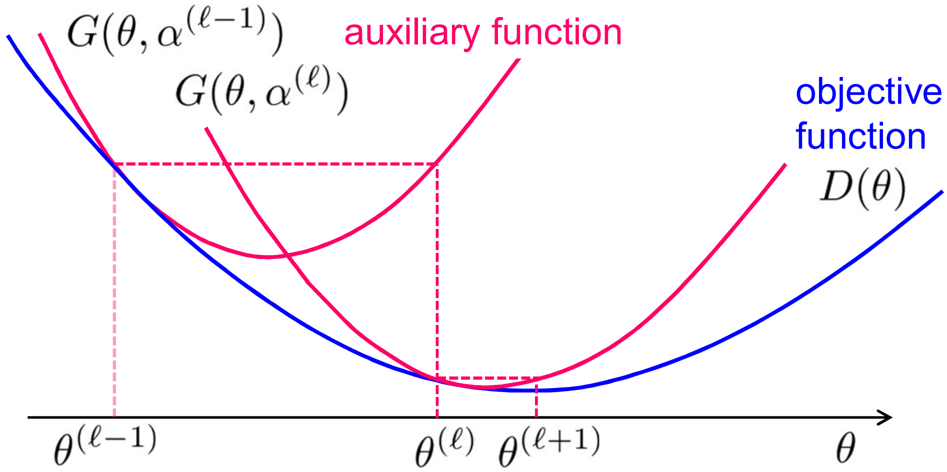
Graphical illustration of MM and ME algorithms

- ME algorithm



Relation to EM algorithm

- Objective: maximize $p(\mathbf{x}|\theta)$ w.r.t. θ

$$\log p(\mathbf{x}|\theta) = \log \int p(\mathbf{x}, \mathbf{c}|\theta) \mathrm{d}\mathbf{c}$$

$\mathbf{c}$: latent variable

$$= \log \int q(\mathbf{c}|\mathbf{x}) \frac{p(\mathbf{x}, \mathbf{c}|\theta)}{q(\mathbf{c}|\mathbf{x})} \mathrm{d}\mathbf{c}$$

$$\geq \int q(\mathbf{c}|\mathbf{x}) \log \frac{p(\mathbf{x}, \mathbf{c}|\theta)}{q(\mathbf{c}|\mathbf{x})} \mathrm{d}\mathbf{c}$$

← Jensen's inequality
Auxiliary function

E step:
Maximize auxiliary function w.r.t. q

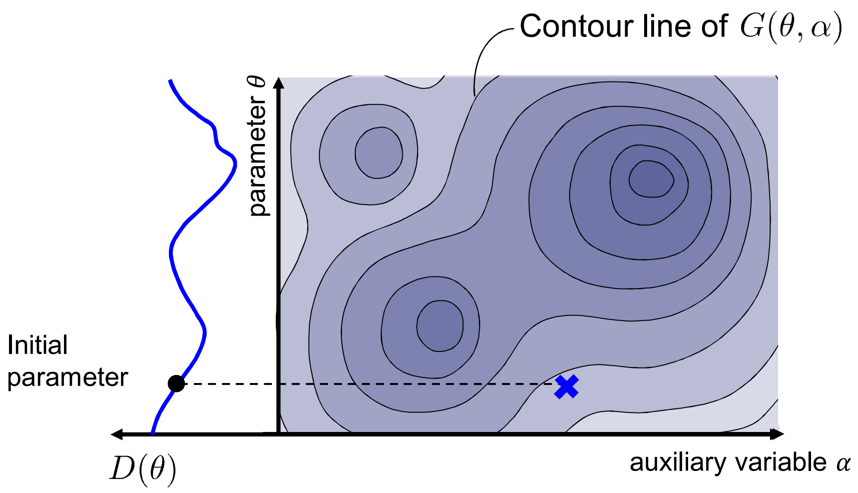
$$q(\mathbf{c}|\mathbf{x}) \leftarrow \frac{p(\mathbf{x}, \mathbf{c}|\theta)}{\int p(\mathbf{x}, \mathbf{c}'|\theta) \mathrm{d}\mathbf{c}'} = p(\mathbf{c}|\mathbf{x}, \theta)$$

M step:
Maximize auxiliary function w.r.t. θ

$$\theta \leftarrow \underset{\theta}{\operatorname{argmax}} \int q(\mathbf{c}|\mathbf{x}) \log \frac{p(\mathbf{x}, \mathbf{c}|\theta)}{q(\mathbf{c}|\mathbf{x})} \mathrm{d}\mathbf{c}$$

Another look at MM algorithm

- Coordinate descent of $G(\theta, \alpha)$



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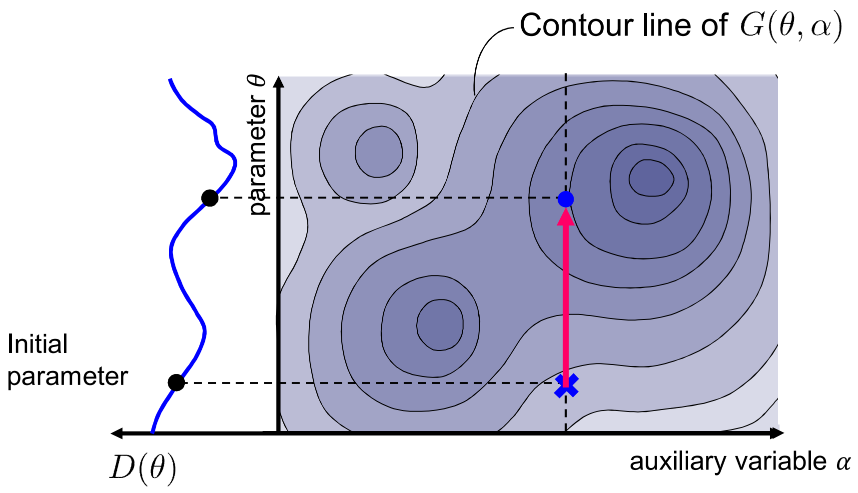
Another look at MM algorithm

- Coordinate descent of $G(\theta, \alpha)$



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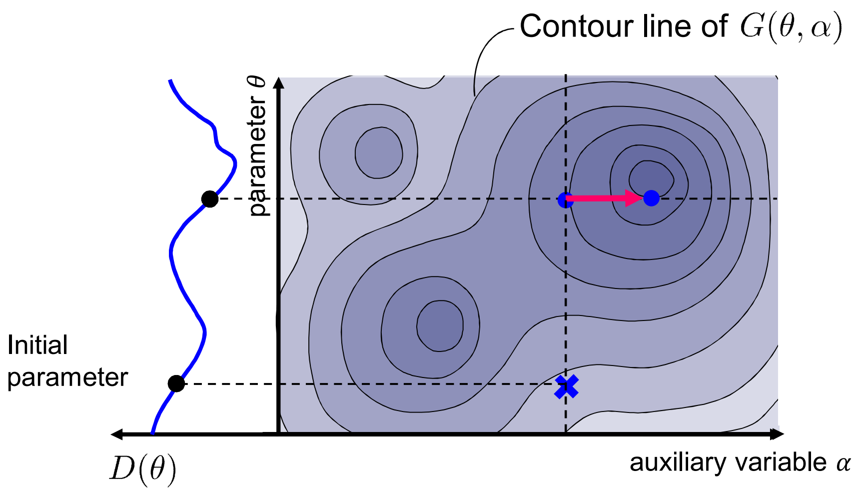
Another look at MM algorithm

- Coordinate descent of $G(\theta, \alpha)$



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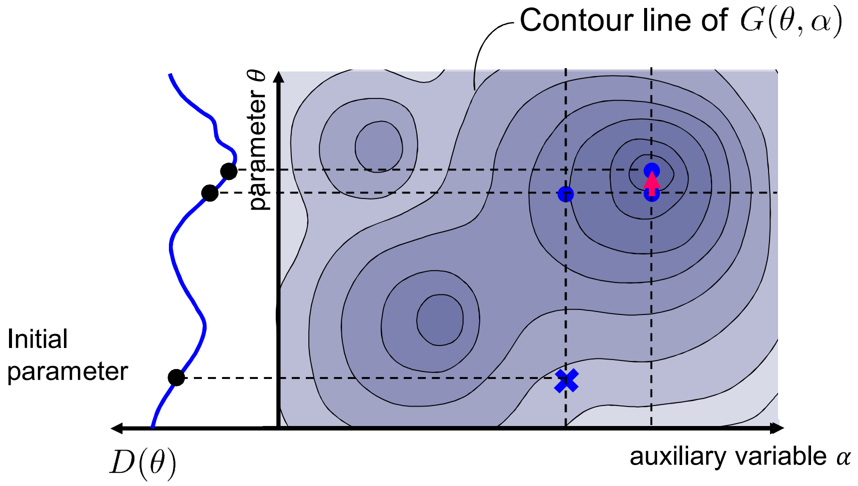
Another look at MM algorithm

- Coordinate descent of $G(\theta, \alpha)$



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Motivations for auxiliary function approach

- Auxiliary function can be useful and effective when one wants to
  - handle a non-convex objective function with multiple local optima and stationary points,
  - handle an objective function that has discontinuous/non-differentiable points,
  - handle equality/inequality constraints, and
  - avoid matrix inversions.

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Useful inequalities for auxiliary function design (1/7)

- Jensen's inequality for non-negative arguments

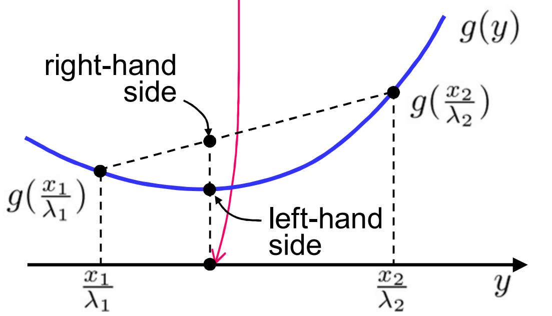
g : convex function
$x_k \geq 0,$
$\lambda_k \geq 0, \quad \sum_k \lambda_k = 1$

$$g\left(\sum_k x_k\right) \leq \sum_k \lambda_k g\left(\frac{x_k}{\lambda_k}\right)$$

Equality holds when $\lambda_k = \dfrac{x_k}{\sum_{k'} x_{k'}}$

E.g.) when $g(y) = -\log y$:

$$-\log\left(\sum_k \lambda_k \frac{x_k}{\lambda_k}\right) \leq -\sum_k \lambda_k \log \frac{x_k}{\lambda_k}$$



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Useful inequalities for auxiliary function design (2/7)

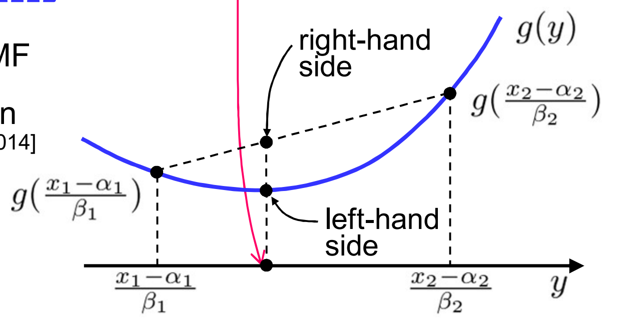
- Jensen's inequality for real number arguments

g : convex function
$\sum_k \alpha_k = 0,$
$\beta_k \geq 0, \quad \sum_k \beta_k = 1$

$$g\left(\sum_k x_k\right) \leq \sum_k \beta_k g\left(\frac{x_k - \alpha_k}{\beta_k}\right)$$

Equality holds when $\alpha_k = x_k - \beta_k\left(\sum_k x_k\right)$

- Used for complex NMF
[Kameoka+2009]
and RBM optimization
[Kameoka+2014][Takamune+2014]



Useful inequalities for auxiliary function design (3/7)

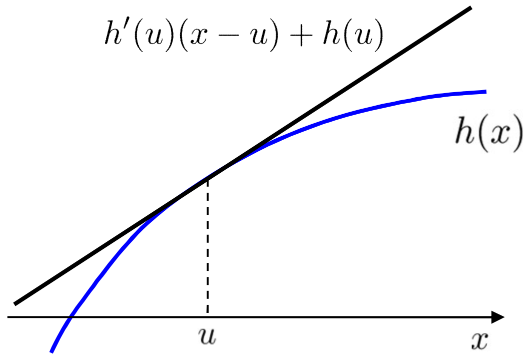
- 1st order Taylor expansion of convex/concave functions

h: concave function → $h(x) \leq h'(u)(x-u) + h(u)$

Equality holds when $u = x$

E.g.) when $h(x) = -\log x$:

$$\log x \leq \frac{1}{u}(x-u) + \log u$$



Useful inequalities for auxiliary function design (4/7)

- 1st order Taylor expansion of logarithmic function

Scalar case:

$$\log x \leq \frac{x}{u} + \log u - 1$$

Equality holds when $u = x$

Extension to matrix case:

$$\log\det \mathbf{X} \leq \mathrm{tr}(\mathbf{U}^{-1}\mathbf{X}) + \log\det \mathbf{U} - M$$

Equality holds when $\mathbf{U} = \mathbf{X}$

→ Equals to the sum of the logarithms of the eigenvalues of $\mathbf{X}$

Used in multichannel NMF frameworks [Sawada+2012, Higuchi+2014] and Positive Semidefinite Tensor Factorization [Yoshii+2013]

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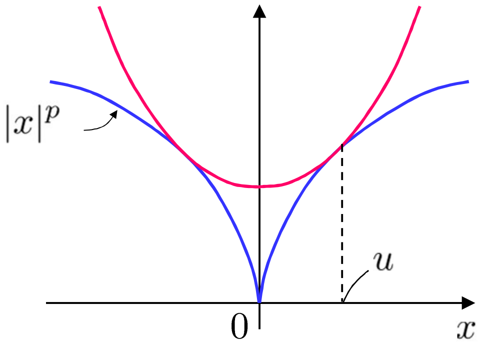
Useful inequalities for auxiliary function design (5/7)

- Quadratic function tangent to power functions

When $0 < p \leq 2$:

$$|x|^p \leq \frac{p}{2}|u|^{p-2}x^2 + |u|^p - \frac{p}{2}|u|^p$$

Equality holds when $u = x$



- Used for sparse regularization for complex NMF [Kameoka+2009]

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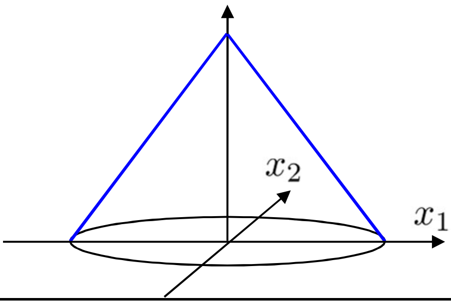
Useful inequalities for auxiliary function design (6/7)

- 1st order Taylor expansion of L2 norm

When $\|\mathbf{w}\|_2 = 1$ → $-\|\mathbf{x}\|_2 \leq -\mathbf{w}^\mathsf{T}\mathbf{x}$ Equality holds when $\mathbf{w} = \mathbf{x}/\|\mathbf{x}\|_2$

For complex number:

When $|\omega| = 1$ → $-|z| \leq -\mathrm{Re}(\omega^* z)$ Equality holds when $\omega = z/|z|$



- Used for sound source localization [Ono+2009][Ono+2010] and Time-domain Spectrogram Factorization (TSF) [Kameoka2015]

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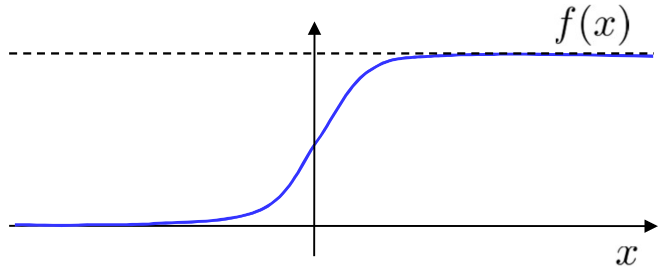
Useful inequalities for auxiliary function design (7/7)

- Logistic function

$$f(x) = \frac{1}{1+\exp(-x)}$$
$$\geq f(\eta)\exp\left(\frac{x-\eta}{2} - \frac{\tanh(\eta/2)}{4\eta}(x^2-\eta^2)\right)$$

Gaussian distribution function

Equality holds when $\eta = x$



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IS-NMF optimization with MM algorithm

[Kameoka+2006][Nakano+2010][Févotte+2011]

- Objective function to be minimized:

$$\mathcal{D}_{\mathrm{IS}}(\mathbf{T},\mathbf{V}) = \sum_{i,j}\left(\frac{|x_{ij}|^2}{\sum_k t_{ik}v_{kj}} + \log\sum_k t_{ik}v_{kj} - \cdots\right)$$

- Reciprocal function is convex in positive domain

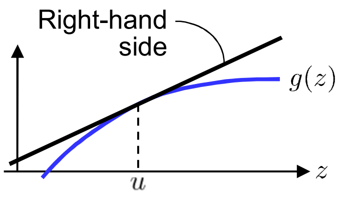
$$\frac{1}{\sum_k z_k} = \frac{1}{\sum_k \lambda_k \frac{z_k}{\lambda_k}} \leq \sum_k \lambda_k \frac{1}{\frac{z_k}{\lambda_k}} = \sum_k \frac{\lambda_k^2}{z_k}$$

Jensen's inequality ⇒ $\frac{1}{\sum_k t_{ik}v_{kj}} \leq \sum_k \frac{\lambda_{ijk}^2}{t_{ik}v_{kj}}$

- Logarithmic function is concave

$$g(u) \leq g'(u)(z-u) + g(u)$$

⇒ $\log\sum_k t_{ik}v_{kj} \leq$
$$\frac{1}{u_{ij}}\left(\sum_k t_{ik}v_{kj} - u_{ij}\right) + \log u_{ij}$$



IS-NMF optimization with MM algorithm

[Kameoka+2006][Nakano+2010][Févotte+2011]

- Majorizer

$$\mathcal{G}_{\mathrm{IS}}(\mathbf{T},\mathbf{V},\boldsymbol{\lambda},\mathbf{U})=\sum_{i,j}\left(\sum_k\frac{\lambda_{ijk}^2|x_{ij}|^2}{t_{ik}v_{kj}}+\frac{1}{u_{ij}}\left(\sum_k t_{ik}v_{kj}-u_{ij}\right)+\log u_{ij}-\cdots\right)$$

- Update rules for $\boldsymbol{\lambda}$ and $\mathbf{U}$

$$\lambda_{ijk}\leftarrow\frac{t_{ik}v_{kj}}{\sum_{k'}t_{ik'}v_{k'j}}\qquad u_{ij}\leftarrow\sum_k t_{ik}v_{kj}$$

- Update rules for $\mathbf{T}$ and $\mathbf{V}$

$$\frac{\partial\mathcal{G}_{\mathrm{IS}}}{\partial t_{ik}}=\sum_j\left(-\frac{\lambda_{ijk}^2|x_{ij}|^2}{t_{ik}^2v_{kj}}+\frac{v_{kj}}{u_{ij}}\right)=0\ \Leftrightarrow t_{ik}=\sqrt{\frac{\sum_j\lambda_{ijk}^2|x_{ij}|^2/v_{kj}}{\sum_j v_{kj}/u_{ij}}}$$

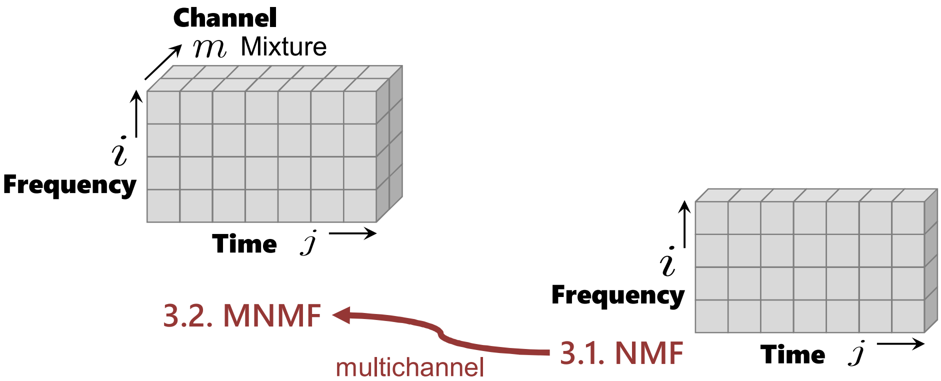
$$\frac{\partial\mathcal{G}_{\mathrm{IS}}}{\partial v_{kj}}=\sum_i\left(-\frac{\lambda_{ijk}^2|x_{ij}|^2}{t_{ik}v_{kj}^2}+\frac{t_{ik}}{u_{ij}}\right)=0\ \Leftrightarrow v_{kj}=\sqrt{\frac{\sum_i\lambda_{ijk}^2|x_{ij}|^2/t_{ik}}{\sum_i t_{ik}/u_{ij}}}$$

* Update rules of ME algorithm can be obtained by solving $\mathbf{T}$ and $\mathbf{V}$ that satisfies $\mathcal{G}_{\mathrm{IS}}(\mathbf{T},\mathbf{V},\boldsymbol{\lambda},\mathbf{U})=\mathcal{G}_{\mathrm{IS}}(\mathbf{T}',\mathbf{V}',\boldsymbol{\lambda},\mathbf{U})$

Tutorial structure

1. **Introduction**
   1. Separation of audio/speech signals
   2. Live demonstration
2. **ICA and IVA**
   1. ICA: Independent Component Analysis
   2. IVA: Independent Vector Analysis
3. **NMF**
   1. NMF: Nonnegative Matrix Factorization
   2. MNMF: Multichannel NMF
4. **ILRMA**
   1. ILRMA: Independent Low-Rank Matrix Analysis

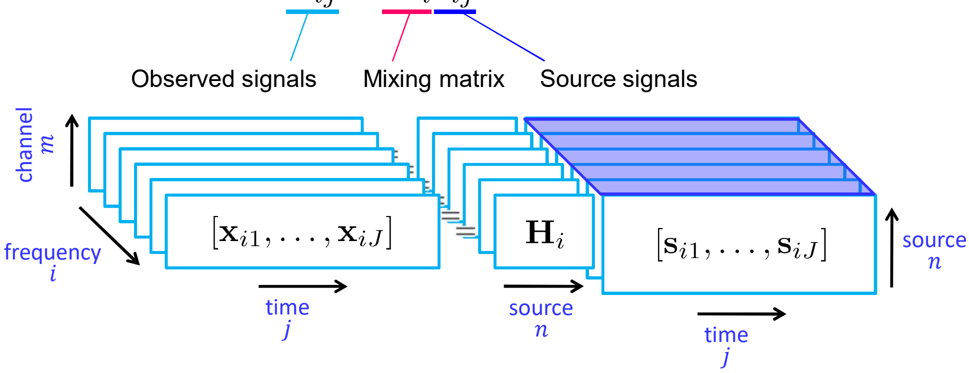
Tensor and sliced matrices



Extension to multichannel input

- Frequency-domain instantaneous mixture:

$$\mathbf{x}_{ij} = \mathbf{H}_i \mathbf{s}_{ij}$$



- Assume local Gaussian model (LGM) with source power spectrograms expressed using NMF (low-rank) model

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Local Gaussian model

- Source model assuming each element of a complex spectrogram to independently follow a zero-mean complex Gaussian distribution with a different variance (power)

$$s_{ij,n} \sim \mathcal{N}_{\mathbb{C}}(s_{ij,n}|0, \sigma_{ij,n}^2)$$

variance

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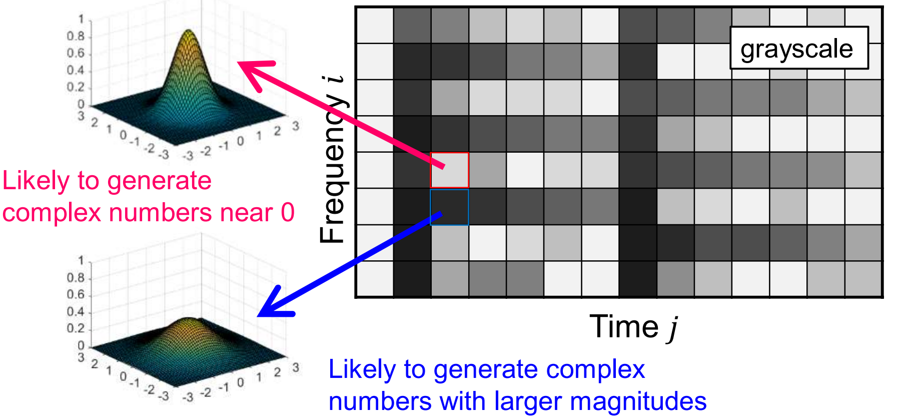
Local Gaussian model

- Source model assuming each element of a complex spectrogram to independently follow a zero-mean complex Gaussian distribution with a different variance (power)

$$s_{ij,n} \sim \mathcal{N}_{\mathbb{C}}(s_{ij,n}|0, \sigma_{ij,n}^2)$$

variance



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Local Gaussian model

- Source model assuming each element of a complex spectrogram to independently follow a zero-mean complex Gaussian distribution with a different variance (power)

$$s_{ij,n} \sim \mathcal{N}_{\mathbb{C}}(s_{ij,n}|0, \sigma^2_{ij,n})$$ variance

  - Allows us to incorporate power spectrogram model in $\sigma^2_{ij,n}$
  - $\sigma^2_{ij,n} = \sum_k t_{ik,n} v_{kj,n}$ corresponds to NMF model

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Local Gaussian model

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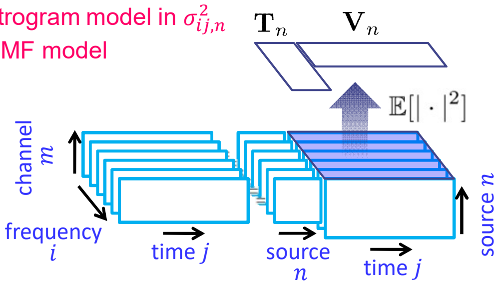
  - $\sigma^2_{ij,n} = \sum_k t_{ik,n} v_{kj,n}$ corresponds to NMF model

$$\mathbf{x}_{ij} = \mathbf{H}_i \mathbf{s}_{ij} \qquad \mathbf{s}_{ij} = \begin{bmatrix} s_{ij,1} \\ \vdots \\ s_{ij,N} \end{bmatrix}$$



Likelihood

$$\mathbf{x}_{ij} \sim \mathcal{N}_{\mathbb{C}}\left(\mathbf{x}_{ij} \middle| \mathbf{0}, \mathbf{H}_i \begin{bmatrix} \sigma^2_{ij,1} & & 0 \\ & \ddots & \\ 0 & & \sigma^2_{ij,N} \end{bmatrix} \mathbf{H}_i^{\mathsf{H}}\right)$$

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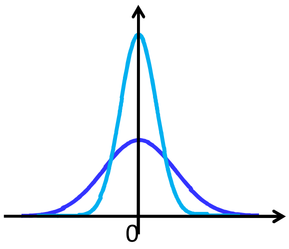
Super-Gaussianity of local Gaussian model

- Theorem

  The time average of zero-mean Gaussian distributions with time-varying variances is super-Gaussian.

  $$p(x) = \sum_n w_n \mathcal{N}(x; 0, v_n) \qquad \left(\sum_n w_n = 1\right)$$

- Proof omitted



Kurtosis becomes 0 if and only if all the variances are equal.

➡ Using local Gaussian models implies assuming source signals to follow super-Gaussian distributions

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Multichannel NMF

[Ozerov&Févotte2010][Kameoka+2010][Sawada+2012]

- Probability density function of microphone array observations

$$\mathbf{x}_{ij} \sim \mathcal{N}_{\mathbb{C}}\left(\mathbf{x}_{ij} \middle| \mathbf{0}, \mathbf{H}_i \begin{bmatrix} \sigma_{ij,1}^2 & & 0 \\ & \ddots & \\ 0 & & \sigma_{ij,N}^2 \end{bmatrix} \mathbf{H}_i^{\mathsf{H}}\right) \longleftarrow \text{local Gaussian model}$$

$$\sum_k t_{ik,n} v_{kj,n} \longleftarrow \text{NMF model}$$

log-likelihood

$$\mathcal{L}(\mathbf{H}, \mathbf{T}, \mathbf{V}) = \sum_{i,j} \{-\log\det \mathbf{\Phi}_{ij} - \mathbf{x}_{ij}^{\mathsf{H}} \mathbf{\Phi}_{ij}^{-1} \mathbf{x}_{ij}\}$$

where $\mathbf{\Phi}_{ij} = \mathbf{H}_i \begin{bmatrix} \sigma_{ij,1}^2 & & 0 \\ & \ddots & \\ 0 & & \sigma_{ij,N}^2 \end{bmatrix} \mathbf{H}_i^{\mathsf{H}}$ and $\sigma_{ij,n}^2 = \sum_k t_{ik,n} v_{kj,n}$

Assumption on mixing matrix
- None [Ozerov+2010][Sawada+2012]
  → Applicable for underdetermined system
- Invertible [Kameoka+2010][Kitamura+2015]
  → Specialized for determined system

Optimization
- EM algorithm [Ozerov+2010][Kameoka+2010]
- Auxiliary function approach [Sawada+2012][Kitamura+2015]

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Multivariate extension of IS divergence

- Different expression of $\boldsymbol{\Phi}_{ij}$

$$\mathbf{x}_{ij} = \mathbf{H}_i \mathbf{s}_{ij} = \sum_n \mathbf{h}_{i,n} \underline{s_{ij,n}}$$

$$s_{ij,n} \sim \mathcal{N}_\mathbb{C}(s_{ij,n} | 0, \sum_k t_{ik,n} v_{kj,n})$$

$$\sim \mathcal{N}_\mathbb{C}\left(\mathbf{x}_{ij} \middle| 0, \sum_n \underbrace{\mathbf{h}_{i,n}\mathbf{h}_{i,n}^\mathsf{H}}_{} \sum_k t_{ik,n} v_{kj,n}\right)$$

$\mathsf{H}_{i,n}$: spatial property of source n

$$\hat{\mathsf{X}}_{ij} (= \boldsymbol{\Phi}_{ij})$$

- Multivariate extension of IS divergence
  (a.k.a. log-determinant divergence)

$$\mathcal{L}(\mathbf{H}, \mathbf{T}, \mathbf{V}) = \sum_{i,j} \{-\log \det \hat{\mathsf{X}}_{ij} - \mathbf{x}_{ij}^\mathsf{H} \hat{\mathsf{X}}_{ij}^{-1} \mathbf{x}_{ij}\}$$

$$= \sum_{i,j} \{-\log \det \underline{\hat{\mathsf{X}}_{ij}} - \mathrm{tr}(\underline{\hat{\mathsf{X}}_{ij}^{-1}} \underline{\mathbf{x}_{ij}\mathbf{x}_{ij}^\mathsf{H}})\}$$

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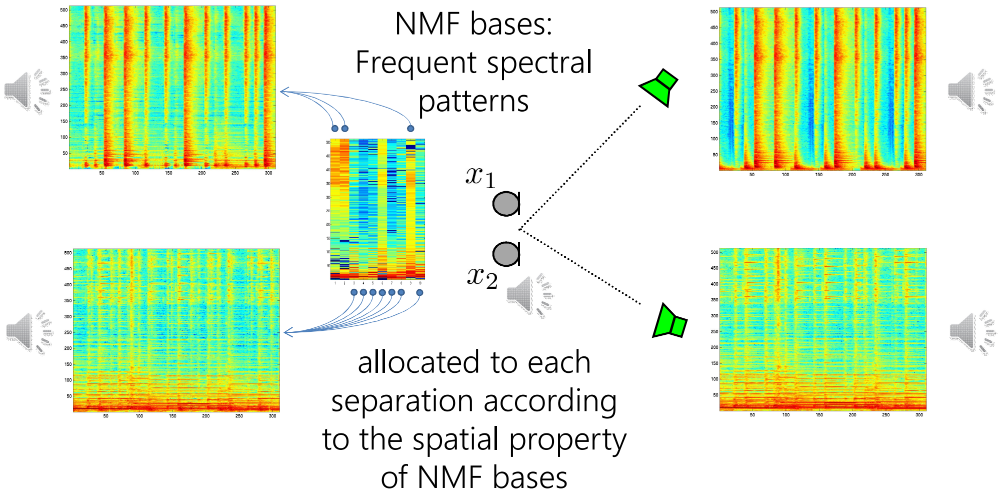
Treating NMF bases as individual sources

Special case where $\hat{\mathsf{X}}_{ij} = \sum_k \mathsf{H}_{ik} t_{ik} v_{kj}$



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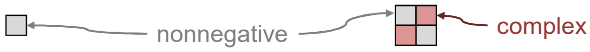
Nonnegativity: from scalar to vector

	Scalar	Vector
Observation	x	$\mathbf{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_M \end{bmatrix}$
Nonnegative representation	$\lvert x \rvert^2 = x x^*$	$\mathsf{X} = \mathbf{x}\mathbf{x}^{\mathsf{H}} = \begin{bmatrix} \lvert x_1 \rvert^2 & \dots & x_1 x_M^* \\ \vdots & \ddots & \vdots \\ x_M x_1^* & \dots & \lvert x_M \rvert^2 \end{bmatrix}$
Low-rank model	$\hat{x}_{ij} = \sum_{k=1}^{K} t_{ik} v_{kj}$	$\hat{\mathsf{X}}_{ij} = \sum_{k=1}^{K} \mathsf{H}_{ik} t_{ik} v_{kj}$



- Hermitian positive-semidefinite matrix
  $\mathsf{H} = \mathsf{H}^H$ All eigenvalues are nonnegative
- Spatial property of the *k*-th NMF basis at frequency *i*

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Optimization problem of multichannel NMF

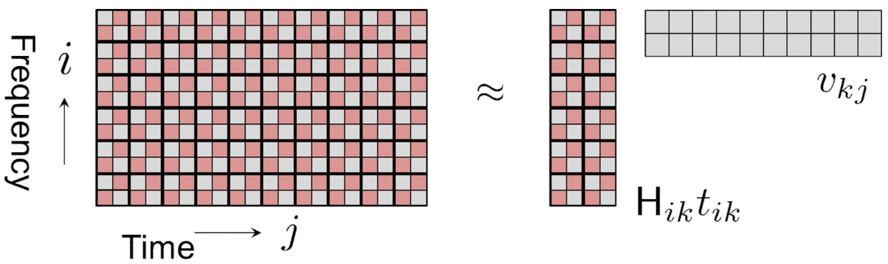
- Objective function to be minimized

$$\mathcal{L}(\mathbf{T}, \mathbf{V}, \mathbf{H}) = \sum_{i,j} d_*(\mathsf{X}_{ij}, \hat{\mathsf{X}}_{ij})$$

typically IS divergence

with $\hat{\mathsf{X}}_{ij} = \sum_{k=1}^{K} \mathsf{H}_{ik} t_{ik} v_{kj}$

$$d_{IS}(\mathsf{X}_{ij}, \hat{\mathsf{X}}_{ij}) = \mathrm{tr}(\mathsf{X}_{ij}\hat{\mathsf{X}}_{ij}^{-1}) - \log\det \mathsf{X}_{ij}\hat{\mathsf{X}}_{ij}^{-1} - M$$



ij-element wise:

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Auxiliary function design

[Sawada+2012, Higuchi+2014, Yoshii+2013]

Objective function

$$\mathcal{L}(\mathbf{T},\mathbf{V},\mathbf{H}) = \sum_{i,j}\left[\mathrm{tr}(\mathsf{X}_{ij}\hat{\mathsf{X}}_{ij}^{-1}) + \log\det\hat{\mathsf{X}}_{ij}\right] \quad \text{with} \quad \hat{\mathsf{X}}_{ij} = \sum_{k=1}^{K}\mathsf{H}_{ik}t_{ik}v_{kj}$$

$$\mathrm{tr}\left[\left(\sum_k \mathbf{X}_k\right)^{-1}\right] \le \sum_k \mathrm{tr}\left(\mathsf{R}_k\mathbf{X}_k^{-1}\mathsf{R}_k\right)$$

matrix extension ⬆

Jensen's inequality applied to reciprocal function

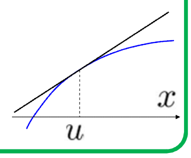
$$\frac{1}{\sum_k x_k} = \frac{1}{\sum_k \lambda_k \frac{x_k}{\lambda_k}} \le \sum_k \frac{\lambda_k^2}{x_k}$$

$$\log\det\hat{\mathsf{X}} \le \mathrm{tr}(\hat{\mathsf{X}}\mathsf{U}^{-1}) + \log\det\mathsf{U} - M$$

matrix extension ⬆

1st order Taylor expansion of logarithmic function

$$\log x \le \frac{x}{u} + \log u - 1$$



Auxiliary function

$$\mathcal{L}^{+}(\mathbf{T},\mathbf{V},\mathbf{H},\mathbf{R},\mathbf{U}) = \sum_{i,j}\left[\sum_k \frac{\mathrm{tr}(\mathsf{X}_{ij}\mathsf{R}_{ijk}\mathsf{H}_{ik}^{-1}\mathsf{R}_{ijk})}{t_{ik}v_{kj}} + \mathrm{tr}\left(\sum_k t_{ik}v_{kj}\mathsf{H}_{ik}\mathsf{U}_{ij}^{-1}\right) + \log\det\mathsf{U}_{ij} - M\right]$$

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Multiplicative update rules

Multichannel IS-NMF

$$t_{ik} \leftarrow t_{ik}\sqrt{\frac{\sum_j v_{kj}\mathrm{tr}(\hat{\mathsf{X}}_{ij}^{-1}\mathsf{X}_{ij}\hat{\mathsf{X}}_{ij}^{-1}\mathsf{H}_{ik})}{\sum_j v_{kj}\mathrm{tr}(\hat{\mathsf{X}}_{ij}^{-1}\mathsf{H}_{ik})}}$$

$$v_{kj} \leftarrow v_{kj}\sqrt{\frac{\sum_i t_{ik}\mathrm{tr}(\hat{\mathsf{X}}_{ij}^{-1}\mathsf{X}_{ij}\hat{\mathsf{X}}_{ij}^{-1}\mathsf{H}_{ik})}{\sum_i t_{ik}\mathrm{tr}(\hat{\mathsf{X}}_{ij}^{-1}\mathsf{H}_{ik})}}$$

$\mathsf{H}_{ik}\mathsf{A}\mathsf{H}_{ik} = \mathsf{B}$ Algebraic Riccati equation

$$\mathsf{A} = \sum_j t_{ik}v_{kj}\hat{\mathsf{X}}_{ij}^{-1}$$

$$\mathsf{B} = \mathsf{H}_{ik}\left(\sum_j t_{ik}v_{kj}\hat{\mathsf{X}}_{ij}^{-1}\mathsf{X}_{ij}\hat{\mathsf{X}}_{ij}^{-1}\right)\mathsf{H}_{ik}$$

IS-NMF

$$t_{ik} \leftarrow t_{ik}\sqrt{\frac{\sum_j \frac{v_{kj}}{\hat{x}_{ij}}\frac{x_{ij}}{\hat{x}_{ij}}}{\sum_j \frac{v_{kj}}{\hat{x}_{ij}}}}$$

$$v_{kj} \leftarrow v_{kj}\sqrt{\frac{\sum_i \frac{t_{ik}}{\hat{x}_{ij}}\frac{x_{ij}}{\hat{x}_{ij}}}{\sum_i \frac{t_{ik}}{\hat{x}_{ij}}}}$$

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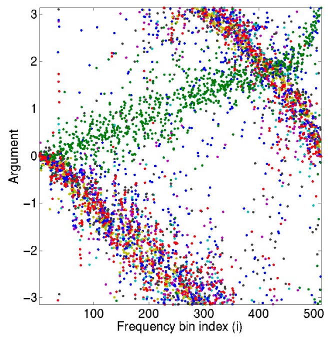
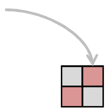
Learned spatial property example

Basis-wise

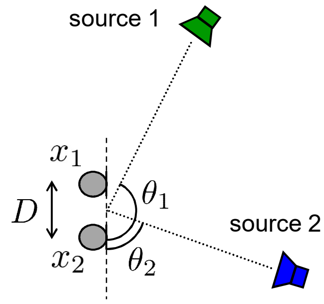


$\arg([\mathsf{H}_{ik}]_{12})$

$k = 1, \ldots, 10$



The 10 bases seem to form 2 clusters, each of which corresponds to each source.



Inter-channel phase difference of source k becomes $\omega_i \frac{D \cos \theta_k}{C}$ where C is speed of sound

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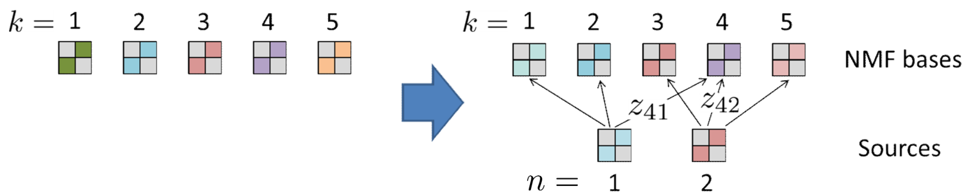
Clustering NMF bases for sources

- Modify the modeling of the spatial property

H_{ik} $\quad\quad$ $\sum_n z_{kn} \mathsf{H}_{in}$



- $\hat{\mathsf{X}}_{ij}$ with source-wise spatial property

$$\hat{\mathsf{X}}_{ij} = \sum_{k=1}^{K} \sum_{n=1}^{N} z_{kn} \mathsf{H}_{in} t_{ik} v_{kj}$$

Similar multiplicative updates derived

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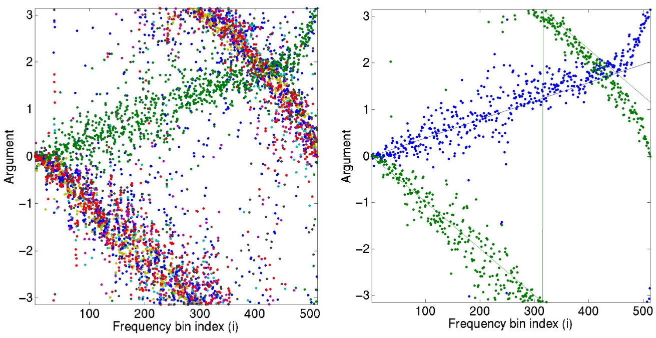
Learned spatial property example

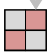
Basis-wise

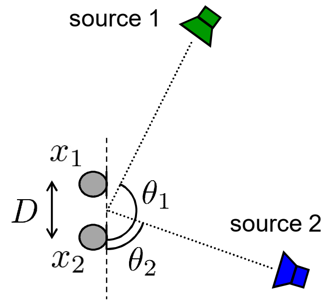
Source-wise



$\arg([\mathsf{H}_{ik}]_{12})$ $\arg([\mathsf{H}_{in}]_{12})$

$k = 1, \ldots, 10$ $n = 1, 2$





Inter-channel phase difference of source k becomes $\omega_i \frac{D \cos \theta_k}{C}$ where C is speed of sound

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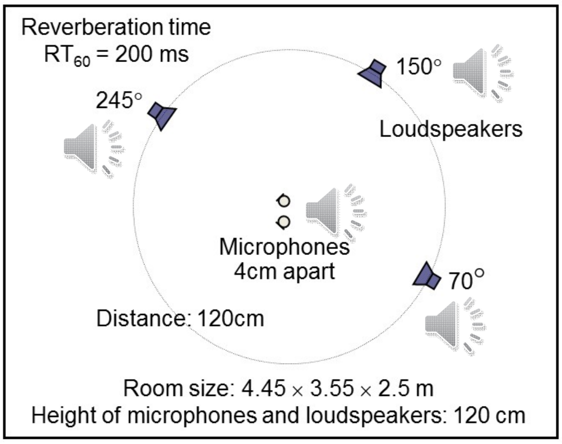
3 music parts separation example

- 3 sources and 2 microphones (underdetermined case)



The computational burden was heavy: it took 838.30 seconds for separating 24-second mixture.

4 examples in total can be found at
http://www.kecl.ntt.co.jp/icl/signal/sawada/demo/mchnmf/

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Determined multichannel NMF

[Kameoka+2010]

- Special case of multichannel NMF where mixing matrix $\mathbf{H}_i$ is invertible
- When $\mathbf{W}_i = \mathbf{H}_i^{-1}$, we can write $\mathbf{W}_i \mathbf{x}_{ij} = \mathbf{s}_{ij}$ and so

$$\mathcal{L}(\mathbf{H}, \mathbf{T}, \mathbf{V}) = \sum_{i,j} \{-\log\det \mathbf{\Phi}_{ij} - \mathbf{x}_{ij}^{\mathsf{H}} \mathbf{\Phi}_{ij}^{-1} \mathbf{x}_{ij}\}$$

$$= \sum_{i,j} \left(\log\det \mathbf{W}_i^{\mathsf{H}} \mathbf{\Sigma}_{ij}^{-1} \mathbf{W}_i - \mathbf{x}_{ij}^{\mathsf{H}} \mathbf{W}_i^{\mathsf{H}} \mathbf{\Sigma}_{ij}^{-1} \mathbf{W}_i \mathbf{x}_{ij} \right)$$

$$= \sum_{i,j} \left\{ 2\log\det \mathbf{W}_i - \sum_n \left(\log \sigma_{ij,n}^2 + \frac{|s_{ij,n}|^2}{\sigma_{ij,n}^2} \right) \right\}$$

where $\mathbf{\Sigma}_{ij} = \begin{bmatrix} \sigma_{ij,1}^2 & & 0 \\ & \ddots & \\ 0 & & \sigma_{ij,N}^2 \end{bmatrix}$ and $\sigma_{ij,n}^2 = \sum_k t_{ik,n} v_{kj,n}$

- Earlier idea of Independent Low-Rank Analysis (ILRMA)

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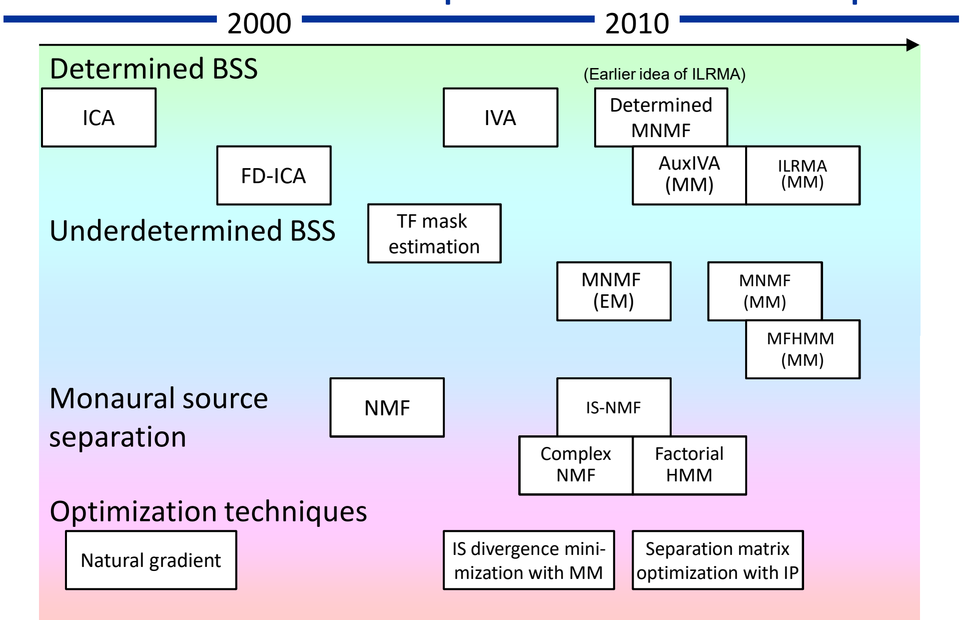
BSS methods and optimization techniques



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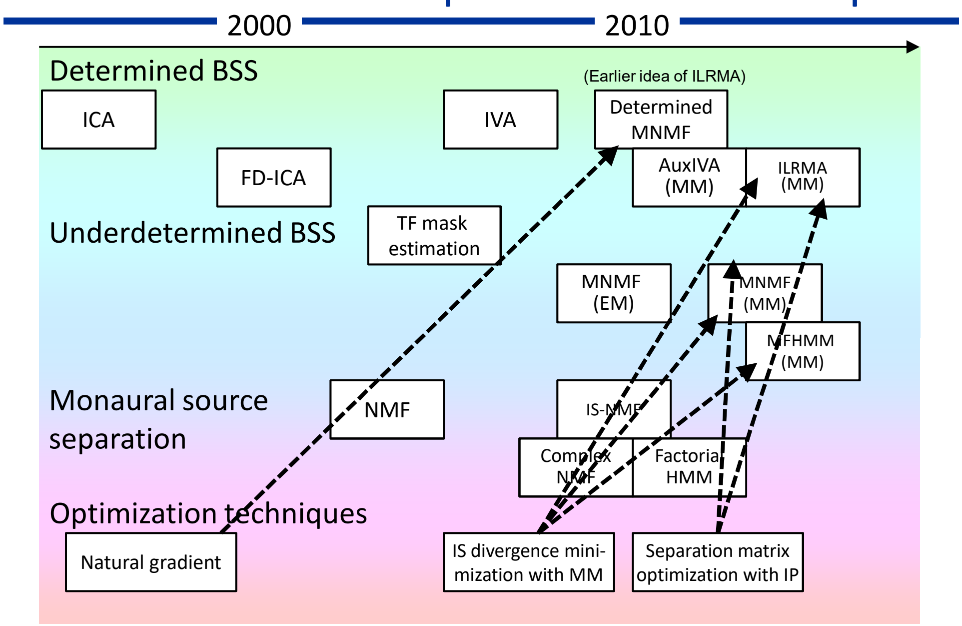
BSS methods and optimization techniques



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Categorization of LGM-based BSS methods

Generative model $\mathbf{x}_{i,j} \sim \mathcal{N}_{\mathbb{C}}(\mathbf{x}_{i,j} | \boldsymbol{\mu}_{i,j}, \boldsymbol{\Phi}_{i,j})$

Method	$\boldsymbol{\mu}_{i,j}$	$\boldsymbol{\Phi}_{i,j}$
Attias (2003)	$\mathbf{0}$	$\sum_n \sum_{j'} \sigma^2_{i,j-j',n} \mathsf{H}_{i,j',n}$
Ozerov & Févotte (2010)	$\mathbf{0}$	$\sum_n \sigma^2_{i,j,n} \mathsf{H}_{i,n} + \mathsf{B}_i$
Duong et al. (2010)	$\mathbf{0}$	$\sum_n \sigma^2_{i,j,n} \mathsf{H}_{i,n}$
Kameoka et al. (2010)	$\mathbf{W}_i^{-1} \sum_{j'=1}^{J'-1} \mathbf{F}_{i,j'} \mathbf{x}_{i,j-j'}$	$\mathbf{W}_i^{-1} \boldsymbol{\Sigma}_{i,j} \mathbf{W}_i^{-1}$
Yoshioka et al. (2011)	$\mathbf{W}_i^{-1} \sum_{j'=1}^{J'-1} \mathbf{F}_{i,j'} \mathbf{x}_{i,j-j'}$	$\mathbf{W}_i^{-1} \boldsymbol{\Sigma}_{i,j} \mathbf{W}_i^{-1}$
Ono et al. (2012)	$\mathbf{0}$	$\mathbf{W}_i^{-1} \boldsymbol{\Sigma}_{i,j} \mathbf{W}_i^{-1}$
Sawada et al. (2013)	$\mathbf{0}$	$\sum_n \sigma^2_{i,j,n} \mathsf{H}_{i,n}$
Higuchi et al. (2014)	$\mathbf{0}$	$\sum_n \sum_{j'} \sigma^2_{i,j-j',n} \mathsf{H}_{i,j',n}$
Kitamura et al. (2015)	$\mathbf{0}$	$\mathbf{W}_i^{-1} \boldsymbol{\Sigma}_{i,j} \mathbf{W}_i^{-1}$
López et al. (2015)	$\mathbf{0}$	$\mathbf{W}_i^{-1} \boldsymbol{\Sigma}_{i,j} \mathbf{W}_i^{-1}$
Adiloğlu & Vincent (2016)	$\mathbf{0}$	$\sum_n \sigma^2_{i,j,n} \mathsf{H}_{i,n}$
Kounades-Bastian et al. (2016)	$\mathbf{0}$	$\sum_n \sigma^2_{i,j,n} \mathsf{H}_{i,j,n}$

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Categorization of LGM-based BSS methods

Constraints on $\sigma^2_{i,j,n}$

Method	$\sigma^2_{i,j,n}$
Attias (2003)	$t_{i,k_{j,n},n}$ where $k_{j,n} \sim \boldsymbol{\pi}$
Ozerov & Févotte (2010)	$\sum_k t_{i,k,n} v_{k,j,n}$
Duong et al. (2010)	$\sigma^2_{i,j,n}$
Kameoka et al. (2010)	$\sum_{k,l} t_{i,k,n} a_{i,l,n} v_{k,j,n}$
Yoshioka et al. (2011)	$v_{j,n} a_{i,j,n}$
Ono et al. (2012)	$v_{j,n}$
Sawada et al. (2013)	$\sum_k z_{k,n} t_{i,k} v_{k,j}$
Higuchi et al. (2014)	$t_{i,k_{j,n},n} v_{j,n}$ where $k_{j,n} \sim \boldsymbol{\pi}$
Kitamura et al. (2015)	$\sum_k z_{k,n} t_{i,k} v_{k,j}$
López et al. (2015)	$\sigma^2_{i,j,n}$
Adiloğlu & Vincent (2016)	$(\sum_k t^{\mathrm{e}}_{i,k,n} v^{\mathrm{e}}_{k,j,n})(\sum_{k'} t^{\mathrm{f}}_{i,k',n} v^{\mathrm{f}}_{k',j,n})$
Kounades-Bastian et al. (2016)	$\sum_k t_{i,k,n} v_{k,j,n}$

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4. **ILRMA**
   1. ILRMA: Independent Low-Rank Matrix Analysis

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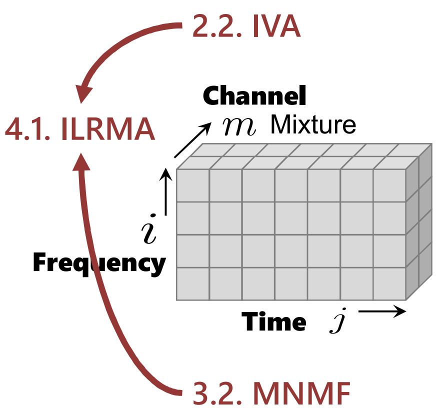
Tensor and sliced matrices



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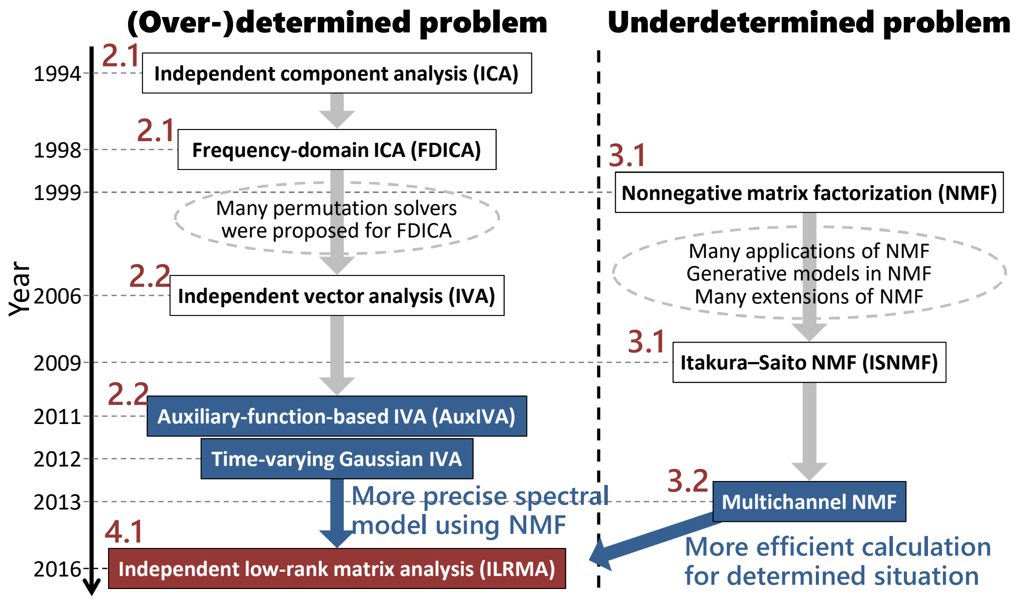
Historical development



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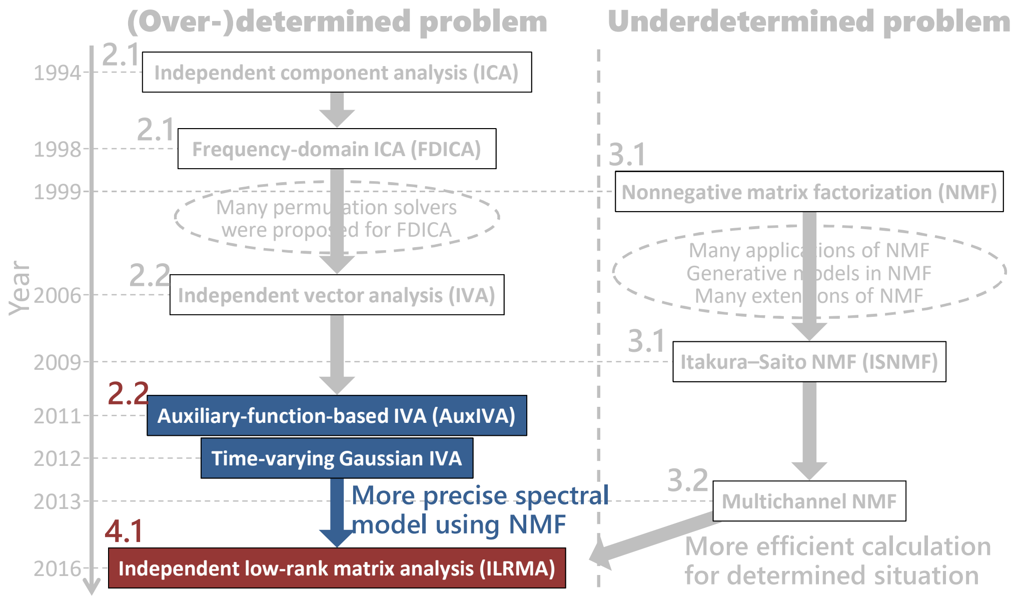
Historical development



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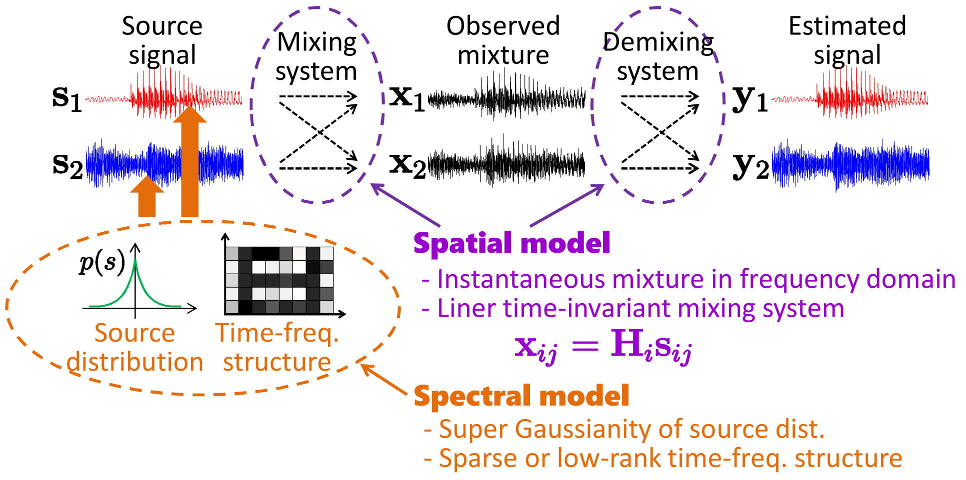
Spatial and spectral models in BSS

- Spatial model: assumption of mixing/demixing system
- Spectral model: assumption for each source



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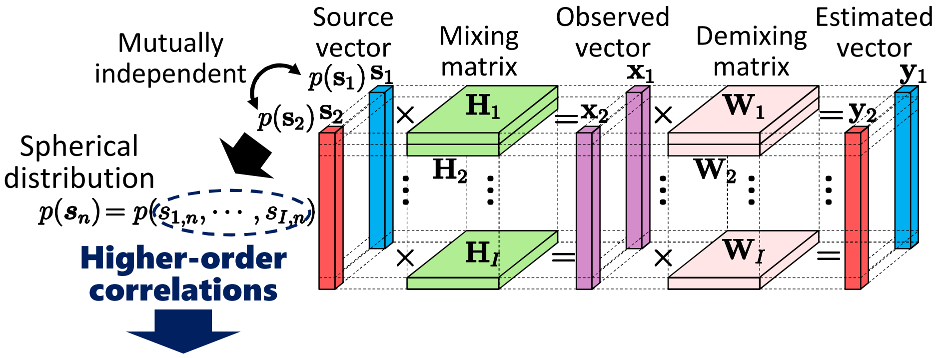
IVA revisited: model

- IVA extends ICA to multivariate probabilistic model

[Hiroe+, 2006], [Kim+, 2006], [Kim+, 2007]

Frequency-domain ICA: frequency **scalar** random variables

IVA: frequency **vector** random variables



Permutation-problem-free estimation of $\mathbf{W}_i$

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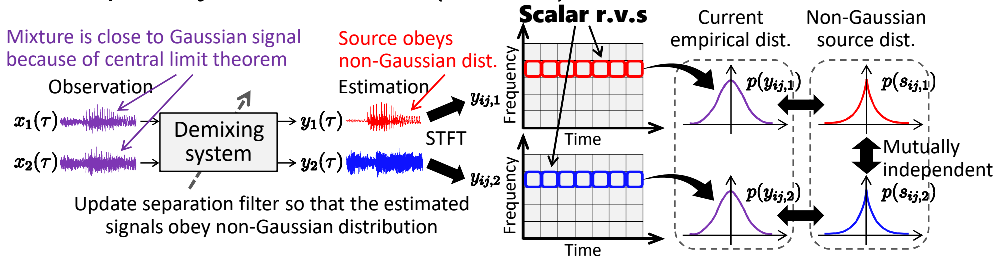
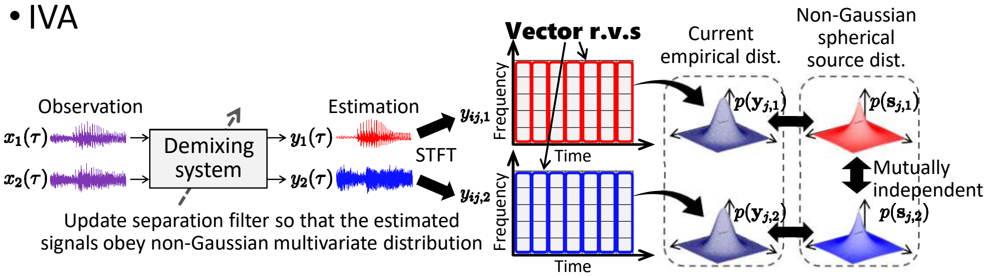
IVA revisited: compared to FDICA

- Frequency-domain ICA (FDICA)



- IVA



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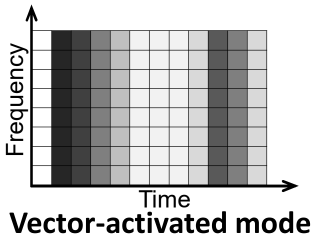
Extension of vector source model in IVA

- Frequency vector spectral model (IVA)

  Co-occurrence among **frequency bins** of each source

Extend vector model to low-rank matrix model



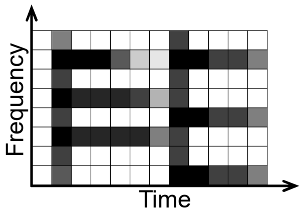
Vector-activated model

- NMF spectral model

  Co-occurrence among **time-frequency slots** of each source with a low-rank structure

More precise representation of time-frequency structure

Incrementation of frequency bases



Low-rank model

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ISNMF revisited: low-rank spectral model

- Itakura–Saito NMF (ISNMF) [Févotte+, 2009]

$$\mathcal{D}_{\mathrm{IS}}\left(|\mathbf{X}|^{\cdot 2} \| \mathbf{TV}\right) = \sum_{i,j}\left[\frac{|x_{ij}|^2}{\sum_k t_{ik}v_{kj}} + \log\sum_k t_{ik}v_{kj}\right]$$

Minimization of the above is equivalent to a maximum likelihood (ML) estimation with a following generative model:

At each time-frequency slot, complex-valued component x_{ij} obeys

Variance

$$x_{ij} \sim \mathcal{N}_c\left(x_{ij}|0, \sum_k t_{ik}v_{kj}\right) = \frac{1}{\pi\sum_k t_{ik}v_{kj}}\exp\left(-\frac{|x_{ij}|^2}{\sum_k t_{ik}v_{kj}}\right)$$

If x_{ij} can be decomposed as $x_{ij} = \sum_k c_{ijk}$, then

$$c_{ijk} \sim \mathcal{N}_c\left(c_{ijk}|0, t_{ik}v_{kj}\right)$$

Parameters are also decomposed

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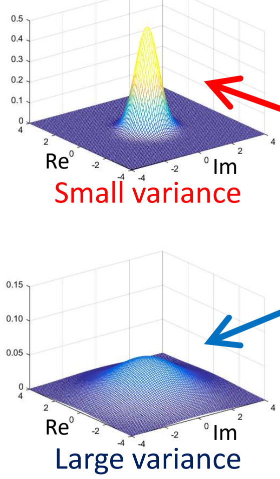
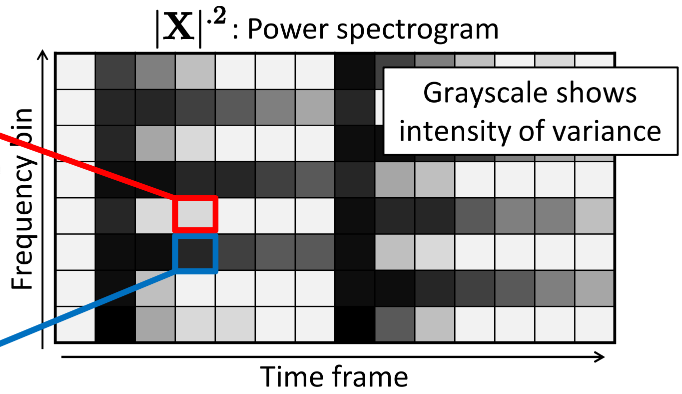
ISNMF revisited: low-rank spectral model

- Itakura–Saito NMF (ISNMF) [Févotte+, 2009]





NOTE: $p(\mathbf{X}) = \prod_{i,j} p(x_{ij}; 0, \sum_k t_{ik} v_{kj})$ is **non-Gaussian** because the variance $\sum_k t_{ik} v_{kj}$ can fluctuate along time and frequency

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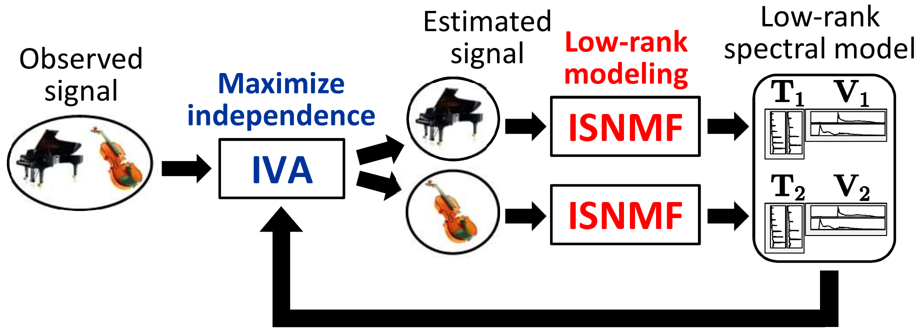
ILRMA: unified method of IVA and NMF

- Independent low-rank matrix analysis (ILRMA) [Kitamura+, 2016]

  Unification of

  1. Estimation of demixing matrix $\mathbf{W}_i$ (**IVA spatial model**)
  2. Low-rank approximation using $\mathbf{T}_n \mathbf{V}_n$ (**ISNMF spectral model**)



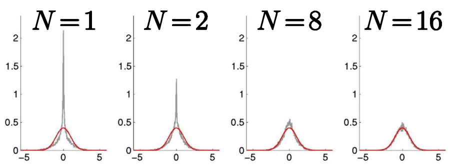
ILRMA: unified method of IVA and NMF

- Independent low-rank matrix analysis (ILRMA) [Kitamura+, 2016]

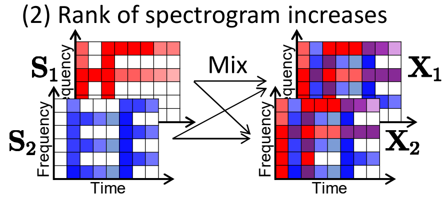
  Motivation of ILRMA?

When sources are mixed…

(1) Mixture dist. approaches to Gaussian



(2) Rank of spectrogram increases



To separate the sources…

Maximize non-Gaussianity
(using ICA or IVA theory)

Restrict rank of separated signal
(using NMF theory)

Low-rank assumption can avoid the permutation problem

Cost function in ILRMA

- Cost function in FDICA, IVA, or ILRMA

Source distribution (spectral model)

$$\mathcal{L} = -2J \sum_i \log |\det \mathbf{W}_i| - \sum_n \log p(\mathbf{Y}_n)$$

FDICA (Laplace)
e.g., [Sawada+, 2003]

$$p(\mathbf{Y}_n) = \prod_{i,j} \alpha \exp(-|y_{ij,n}|)$$

Separable for frequency

IVA (spherical Laplace)
[Hiroe+, 2006], [Kim+, 2006]

$$p(\mathbf{Y}_n) = \prod_j \alpha \exp(-\|\mathbf{y}_{j,n}\|_2)$$

$$\mathbf{y}_{j,n} = \begin{pmatrix} y_{1j,n} \\ \vdots \\ y_{Ij,n} \end{pmatrix}$$

Frequency vector

IVA (time-varying Gauss)
[Ono+, 2012]

$$p(\mathbf{Y}_n) = \prod_j \frac{\alpha}{\sigma_{j,n}^2} \exp\left(-\frac{\|\mathbf{y}_{j,n}\|_2^2}{\sigma_{j,n}^2}\right)$$

ILRMA (ISNMF model)
[Kitamura+, 2016]

$$p(\mathbf{Y}_n) = \prod_{i,j} \frac{1}{\pi \sum_k t_{ik,n} v_{kj,n}} \exp\left(-\frac{|y_{ij,n}|^2}{\sum_k t_{ik,n} v_{kj,n}}\right)$$

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Cost function in ILRMA

- Cost function in ILRMA

Demixing matrix: $\mathbf{W}_i = (\mathbf{w}_{i,1} \cdots \mathbf{w}_{i,N})^{\mathrm{H}}$ Estimated signal: $y_{ij,n} = \mathbf{w}_{i,n}^{\mathrm{H}}\mathbf{x}_{ij}$

$$\mathcal{L} = -2J\sum_i \log|\det \mathbf{W}_i| + \sum_{i,j,n}\left[\frac{|y_{ij,n}|^2}{\sum_k t_{ik,n}v_{kj,n}} + \log\sum_k t_{ik,n}v_{kj,n}\right]$$

Cost function in **time-varying Gaussian IVA** (estimates demixing matrix)

Cost function in **ISNMF** (estimates low-rank spectral structure)

Spatial model update: AuxIVA
Spectral model update: ISNMF
➡ All the variables are alternatively updated

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Update rule of parameters in ILRMA

- Maximum-likelihood-based update rules

Alternatively update both models

Spatial model (demixing matrix)

Update demixing filter [Ono+, 2011]

$$\mathbf{V}_{i,n} = \frac{1}{J}\sum_j \frac{1}{r_{ij,n}}\mathbf{x}_{ij}\mathbf{x}_{ij}^{\mathrm{H}}$$

$$\mathbf{w}_{i,n} \leftarrow (\mathbf{W}_i\mathbf{V}_{i,n})^{-1}\mathbf{e}_n$$

$$\mathbf{w}_{i,n} \leftarrow \mathbf{w}_{i,n}\left(\mathbf{w}_{i,n}^{\mathrm{H}}\mathbf{V}_{i,n}\mathbf{w}_{i,n}\right)^{-\frac{1}{2}}$$

Update estimated signal

$$y_{ij,n} = \mathbf{w}_{i,n}^{\mathrm{H}}\mathbf{x}_{ij}$$

Spectral model (NMF variables)

Update NMF parameters [Nakano+, 2010]

$$t_{ik,n} \leftarrow t_{ik,n}\sqrt{\frac{\sum_j |y_{ij,n}|^2 v_{kj,n}\left(\sum_{k'} t_{ik',n}v_{k'j,n}\right)^{-2}}{\sum_j v_{kj,n}\left(\sum_{k'} t_{ik',n}v_{k'j,n}\right)^{-1}}}$$

$$v_{kj,n} \leftarrow v_{kj,n}\sqrt{\frac{\sum_j |y_{ij,n}|^2 t_{ik,n}\left(\sum_{k'} t_{ik',n}v_{k'j,n}\right)^{-2}}{\sum_j t_{ik,n}\left(\sum_{k'} t_{ik',n}v_{k'j,n}\right)^{-1}}}$$

Update estimated variance

$$r_{ij,n} = \sum_k t_{ik,n}v_{kj,n}$$

$\mathbf{e}_n$: one-hot vector with one at nth element

See also (pseudo code)
http://d-kitamura.net/pdf/misc/AlgorithmsForIndependentLowRankMatrixAnalysis.pdf

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Clustering NMF bases for sources

- NMF bases should be automatically clustered into each source

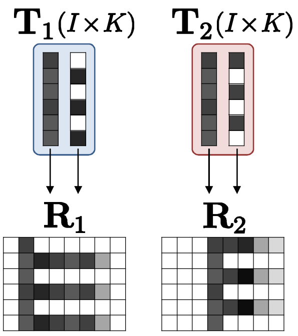
$$r_{ij,n} = \sum_k t_{ik,n} v_{kj,n}$$

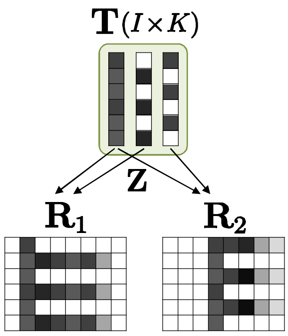
Fixed number of bases for each source

Introduce partitioning function

$$r_{ij,n} = \sum_k z_{kn} t_{ik} v_{kj}$$

Adaptive number of bases for each source





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Update rule of parameters in ILRMA

- Maximum-likelihood-based update rules with $\mathbf{Z}$

Alternatively update both models

Spatial model
(demixing matrix)

Update demixing filter [Ono+, 2011]

$$\mathbf{V}_{i,n} = \frac{1}{J} \sum_j \frac{1}{r_{ij,n}} \mathbf{x}_{ij} \mathbf{x}_{ij}^{\mathrm{H}}$$

$$\mathbf{w}_{i,n} \leftarrow (\mathbf{W}_i \mathbf{V}_{i,n})^{-1} \mathbf{e}_n$$

$$\mathbf{w}_{i,n} \leftarrow \mathbf{w}_{i,n} \left(\mathbf{w}_{i,n}^{\mathrm{H}} \mathbf{V}_{i,n} \mathbf{w}_{i,n} \right)^{-\frac{1}{2}}$$

Update estimated signal

$$y_{ij,n} = \mathbf{w}_{i,n}^{\mathrm{H}} \mathbf{x}_{ij}$$

$\mathbf{e}_n$: one-hot vector with one at nth element

Spectral model
(NMF variables)

Update NMF parameters [Nakano+, 2010]

$$z_{kn} \leftarrow z_{kn} \sqrt{\frac{\sum_{i,j} |y_{ij,n}|^2 t_{ik} v_{kj} \left(\sum_{k'} z_{k'n} t_{ik'} v_{k'j} \right)^{-2}}{\sum_{i,j} t_{ik} v_{kj} \left(\sum_{k'} z_{k'n} t_{ik'} v_{k'j} \right)^{-1}}}$$

$$t_{ik} \leftarrow t_{ik} \sqrt{\frac{\sum_{j,n} |y_{ij,n}|^2 z_{kn} v_{kj} \left(\sum_{k'} z_{k'n} t_{ik'} v_{k'j} \right)^{-2}}{\sum_{j,n} z_{kn} v_{kj} \left(\sum_{k'} z_{k'n} t_{ik'} v_{k'j} \right)^{-1}}}$$

$$v_{kj} \leftarrow v_{kj} \sqrt{\frac{\sum_{i,n} |y_{ij,n}|^2 z_{kn} t_{ik} \left(\sum_{k'} z_{k'n} t_{ik'} v_{k'j} \right)^{-2}}{\sum_{i,n} z_{kn} t_{ik} \left(\sum_{k'} z_{k'n} t_{ik'} v_{k'j} \right)^{-1}}}$$

Update estimated variance

$$r_{ij,n} = \sum_k z_{kn} t_{ik} v_{kj}$$

See also (pseudo code)
http://d-kitamura.net/pdf/misc/AlgorithmsForIndependentLowRankMatrixAnalysis.pdf

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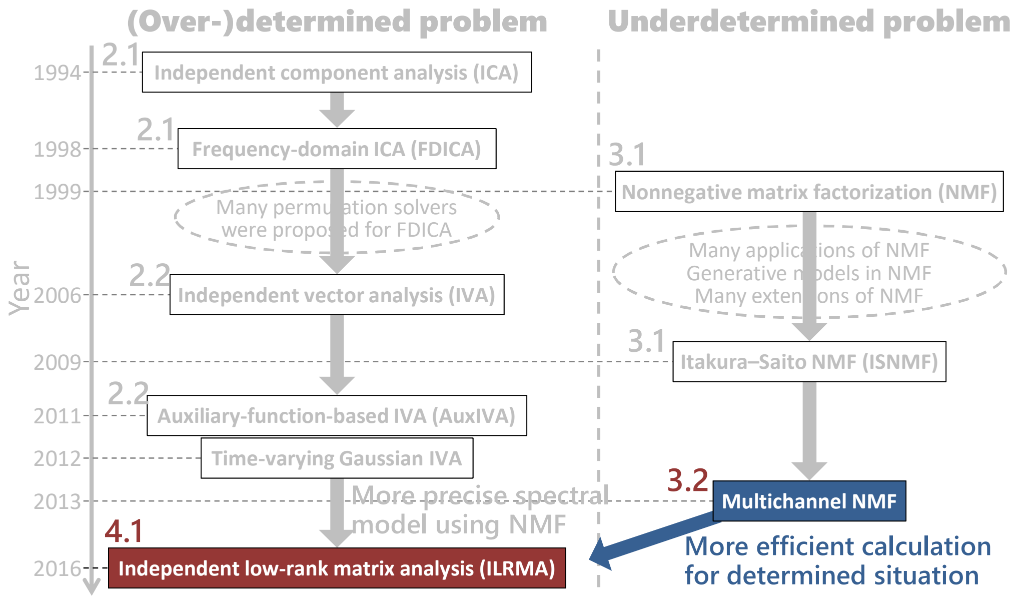
Historical development



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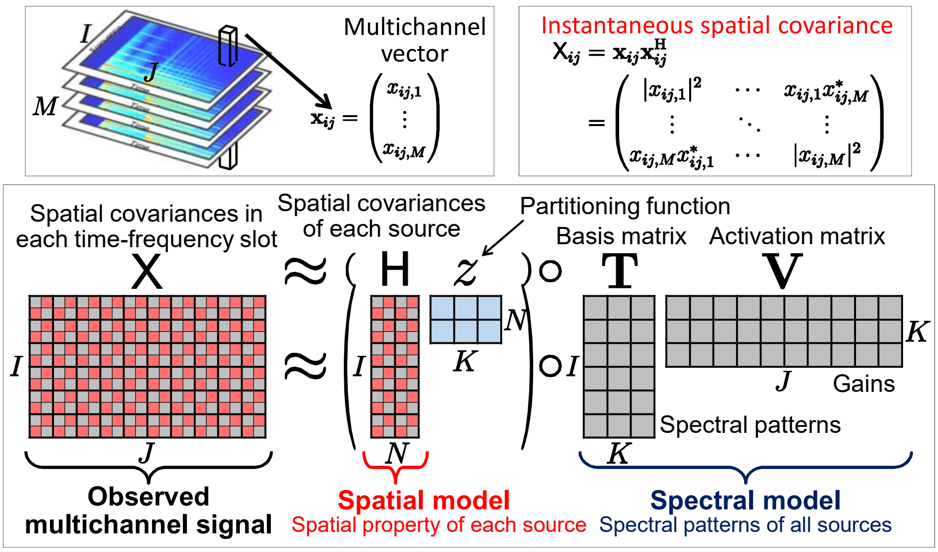
Multichannel NMF revisited: model

- Multichannel NMF with full-rank covariance [Sawada+, 2013]



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ILRMA and multichannel ISNMF

- Relationship b/w ILRMA and multichannel ISNMF?

  Source distribution: **same** $p(y_{ij,n}) = \mathcal{N}_c(y_{ij,n}; 0, \sum_k t_{ik,n} v_{kj,n})$

  Spatial model: **different**

ILRMA	**Multichannel ISNMF**
Instantaneous mixture in frequency domain	Mixture of full-rank covariances (and power spectrograms)
$\mathbf{x}_{ij} = \mathbf{H}_i \mathbf{s}_{ij}$	$\hat{\mathsf{X}}_{ij} = \sum_n \sum_k z_{kn} t_{ik} v_{kj} \mathsf{H}_{i,n}$

- Rank-1 spatial model [Duong+, 2010]

  Equivalent to instantaneous mixture

$$\mathsf{H}_{i,n} = \mathbf{h}_{i,n} \mathbf{h}_{i,n}^{\mathrm{H}}$$

$\mathbf{h}_{i,n}$: steering vector (column vector of mixing matrix)

$\mathbf{H}_i = (\mathbf{h}_{i,1} \ \cdots \ \mathbf{h}_{i,N})$

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ILRMA and multichannel ISNMF

- Multichannel ISNMF with rank-1 spatial model

  Cost function of multichannel ISNMF

$$\mathcal{J} = \sum_{i,j} \left[\mathrm{tr}\left(\mathsf{X}_{ij} \hat{\mathsf{X}}_{ij}^{-1} \right) + \log \det \hat{\mathsf{X}}_{ij} \right]$$

$$\hat{\mathsf{X}}_{ij} = \sum_k \sum_n \mathsf{H}_{i,n} z_{kn} t_{ik} v_{kj}$$

Substitute rank-1 spatial model $\mathsf{H}_{i,n} = \mathbf{h}_{i,n} \mathbf{h}_{i,n}^{\mathrm{H}}$ into $\hat{\mathsf{X}}_{ij}$

$$\begin{aligned} \hat{\mathsf{X}}_{ij} &= \sum_k \sum_n \mathbf{h}_{i,n} \mathbf{h}_{i,n}^{\mathrm{H}} z_{kn} t_{ik} v_{kj} \\ &= \sum_n \mathbf{h}_{i,n} \mathbf{h}_{i,n}^{\mathrm{H}} \sum_k z_{kn} t_{ik} v_{kj} \\ &= \mathbf{H}_i \mathbf{D}_{ij} \mathbf{H}_i^{\mathrm{H}} \end{aligned}$$

$$\mathbf{H}_i = (\mathbf{h}_{i,1} \ \cdots \ \mathbf{h}_{i,N}), \ \mathbf{D}_{ij} = \begin{pmatrix} \sum_k z_{k1} t_{ik} v_{kj} & 0 & \cdots & 0 \\ 0 & \sum_k z_{k2} t_{ik} v_{kj} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \sum_k z_{kN} t_{ik} v_{kj} \end{pmatrix}$$

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ILRMA and multichannel ISNMF

- Multichannel ISNMF with rank-1 spatial model

Substitute $\hat{\mathrm{X}}_{ij} = \mathbf{H}_i \mathbf{D}_{ij} \mathbf{H}_i^{\mathrm{H}}$ into the cost function of multichannel ISNMF

$$\mathcal{J} = \sum_{i,j} \left[\mathrm{tr}\left(\mathbf{x}_{ij}\mathbf{x}_{ij}^{\mathrm{H}} (\mathbf{H}_i \mathbf{D}_{ij} \mathbf{H}_i^{\mathrm{H}})^{-1} \right) + \log\det \mathbf{H}_i \mathbf{D}_{ij} \mathbf{H}_i^{\mathrm{H}} \right]$$
$$= \sum_{i,j} \left[\mathrm{tr}\left(\mathbf{x}_{ij}\mathbf{x}_{ij}^{\mathrm{H}} \mathbf{H}_i^{-\mathrm{H}} \mathbf{D}_{ij}^{-1} \mathbf{H}_i^{-1} \right) + \log(\det \mathbf{H}_i)(\det \mathbf{D}_{ij})(\det \mathbf{H}_i)^{\mathrm{H}} \right]$$

Transform the variables as $\mathbf{W}_i = \mathbf{H}^{-1}$ and $\mathbf{y}_{ij} = \mathbf{W}_i \mathbf{x}_{ij}$

$$\mathcal{J} = \sum_{i,j} \left[\mathrm{tr}\left(\mathbf{W}_i^{-1} \mathbf{y}_{ij}\mathbf{y}_{ij}^{\mathrm{H}} \mathbf{W}_i^{-\mathrm{H}} \mathbf{W}_i^{\mathrm{H}} \mathbf{D}_{ij}^{-1} \mathbf{W}_i \right) + \log|\det \mathbf{H}_i|^2 + \log\det \mathbf{D}_{ij} \right]$$
$$= -2J \sum_i \log|\det \mathbf{W}_i| + \sum_{i,j} \left[\log \prod_n \sum_k z_{kn} t_{ik} v_{kj} + \mathrm{tr}\left(\mathbf{y}_{ij}\mathbf{y}_{ij}^{\mathrm{H}} \mathbf{D}_{ij}^{-1} \right) \right]$$
$$= -2J \sum_i \log|\det \mathbf{W}_i| + \sum_{i,j,n} \left[\sum_k z_{kn} t_{ik} v_{kj} + \frac{|y_{ij,n}|^2}{\sum_k z_{kn} t_{ik} v_{kj}} \right]$$

Cost in ILRMA

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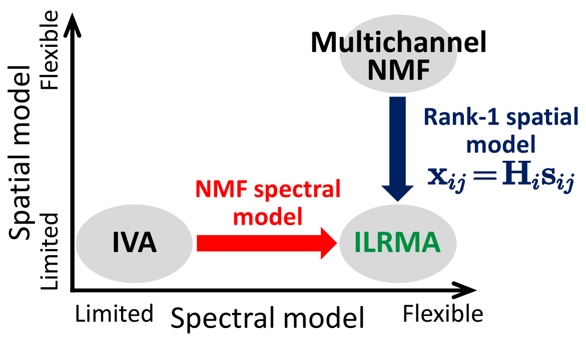
Summary of ILRMA

- From IVA side:

Introduce NMF spectral model (basis incrementation)

- From multichannel NMF side:

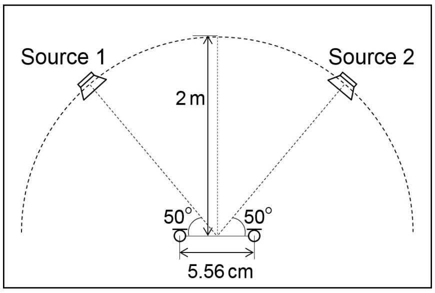
Introduce rank-1 spatial model (instantaneous mixture assumption)



Experiment

- Conditions

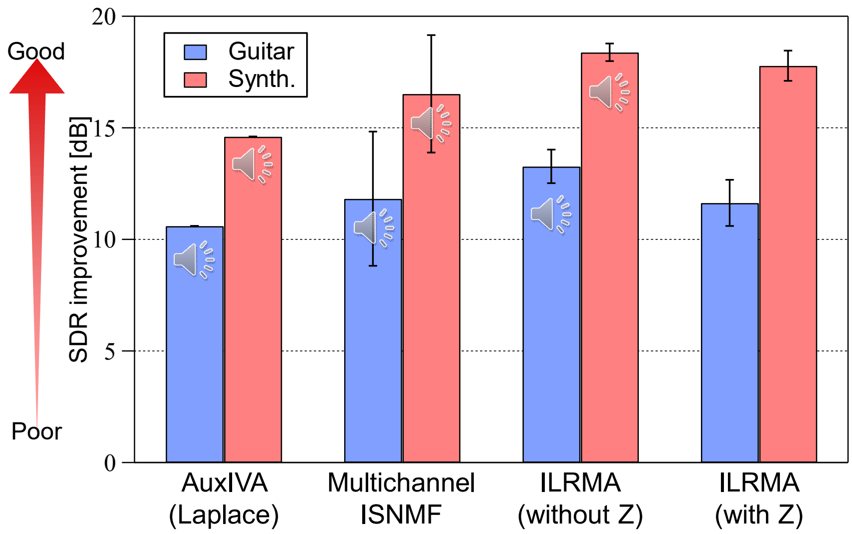
Source signals	Music signals obtained from SiSEC2011 Two microphones and two sources (determined)
Analysis window	512-ms-long Hamming window
Shift length	128 ms (1/4 shift)
Number of bases	30 per each source/60 for all sources
Evaluation score	Improvement ot signal-to-distortion ratio (SDR)



Impulse response E2A
(reverberation time: 300 ms)

Experiment

- Two source case (ultimate nz tour)



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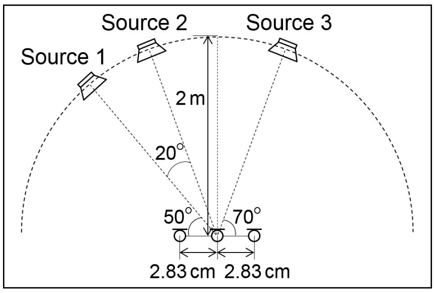
Experiment

- Conditions

Source signals	Music signals obtained from SiSEC2011 Three microphones and three sources (determined)
Analysis window	512-ms-long Hamming window
Shift length	128 ms (1/4 shift)
Number of bases	30 per each source/90 for all sources
Evaluation score	Improvement ot signal-to-distortion ratio (SDR)



Impulse response E2A
(reverberation time: 300 ms)

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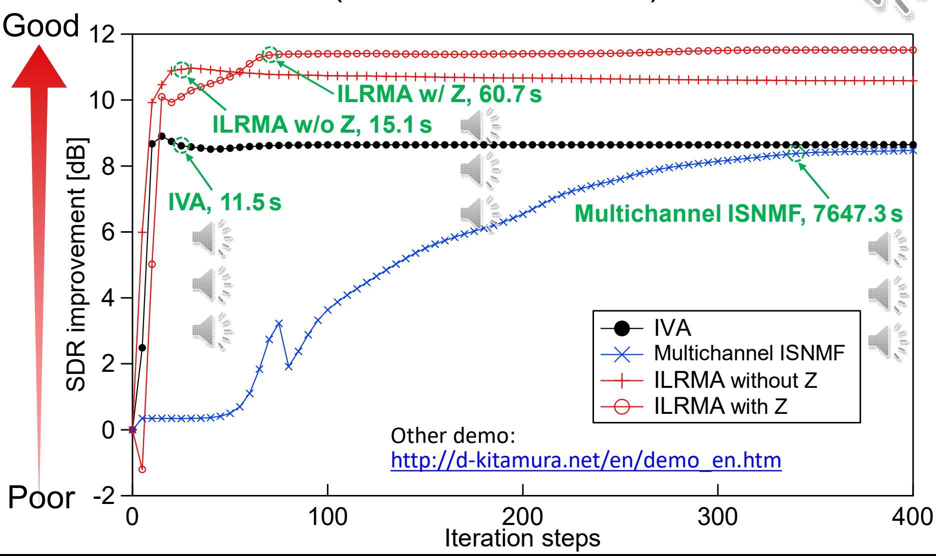
Experiment

- Three source case (bearlin-roads, 14 s)



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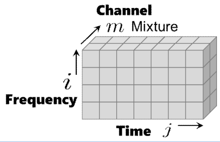
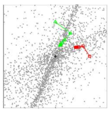
Conclusion

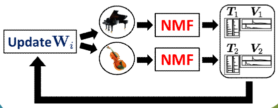
Complex-valued Tensor
- Multichannel audio mixture
- Frequency domain via STFT



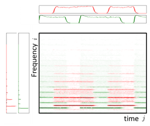
ICA, IVA
- Independence
- Super-Gaussian



ILRMA

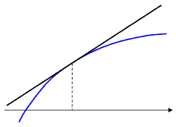


NMF, MNMF
- Spectral bases
- Low-rank



Component → Vector → Low-rank Matrix

Auxiliary function-based optimization
- Fast convergence
- Problem dependent, but "cookbook" works



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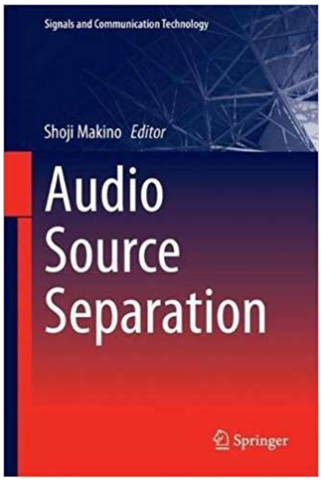
Advertisement: book chapters

- MNMF and ILRMA will be published from Springer in March, 2018!



Audio Source Separation (Signals and Communication Technology)
1st ed. 2018 by Ed. Shoji Makino

Ch. 5
General formulation of multichannel extensions of NMF variants;
Hirokazu Kameoka, Hiroshi Sawada, and Takuya Higuchi.

Ch. 6
Determined Blind Source Separation with Independent Low-Rank Matrix analysis;
Daichi Kitamura, Nobutaka Ono, Hiroshi Sawada, Hirokazu Kameoka, and Hiroshi Saruwatari.

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Thank you very much
for attending this tutorial !

Hiroshi Sawada

Nobutaka Ono

Hirokazu Kameoka

Daichi Kitamura

Selected references

Frequency-domain approach to convolutive BSS

- [Smaragdis, 1998], [Parra and Spence, 2000], [Schobben and Sommen, 2002], [Anemüller and Kollmeier, 2000], [Asano et al., 2003], [Saruwatari et al., 2003], [Saruwatari et al., 2006], [Yoshioka et al., 2008], [Duong et al., 2010]
- Permutation alignment [Sawada et al., 2004], [Sawada et al., 2007], [Sawada et al., 2011]
- Scaling adjustment to microphone observations [Cardoso, 1998], [Murata et al., 2001], [Matsuoka and Nakashima, 2001], [Takatani et al., 2004], [Mori et al., 2006]

ICA: Independent Component Analysis

- Information-maximization approach [Bell and Sejnowski, 1995]
- Maximum likelihood (ML) estimation [Cardoso, 1997]
- Natural gradient [Amari et al., 1996], [Cichocki and Amari, 2002]
- Equivariance property [Cardoso and Souloumiac, 1996]
- FastICA [Hyvärinen et al., 2001]
- Complex-valued ICA [Bingham and Hyvärinen, 2000], [Sawada et al., 2003]
- Auxiliary function based ICA [Ono and Miyabe, 2010]

IVA: Independent Vector Analysis

- Multivariate p.d.f. [Hiroe, 2006], [Kim et al., 2006], [Kim et al., 2007]
- FastIVA [Lee et al., 2006], [Lee et al., 2007]
- HEAD problem [Yeredor, 2009]
- Auxiliary function based IVA (AuxIVA) [Ono, 2011], [Ono, 2012b], [Ikeshita et al., 2017]
- Time-varying Gaussian p.d.f. [Ono, 2012a], [Ono et al., 2012]
- Supervised or model-based IVA [Ono et al., 2012], [Lopez et al., 2015], [Nesta and Koldovský, 2017]
- Online IVA [Kim, 2010], [Taniguchi et al., 2014], [Sunohara et al., 2017]

NMF: Nonnegative Matrix Factorization

- Auxiliary function based optimization for Euclidean distance NMF and generalized Kullback-Leibler divergence NMF [Lee and Seung, 1999], [Lee and Seung, 2001]
- EM-based optimization for Itakura-Saito divergence NMF [Févotte et al., 2009]
- Auxiliary function based optimization for Itakura-Saito divergence NMF [Kameoka et al., 2006], [Nakano et al., 2010], [Févotte and Idier, 2011]
- Auxiliary function based optimization for β divergence NMF [Nakano et al., 2010], [Févotte and Idier, 2011]
- Auxiliary function based optimization for sparse NMF [Kameoka et al., 2009]

Multi-channel NMF

- EM-based optimization [Ozerov and Févotte, 2010]
- Auxiliary function based optimization [Sawada et al., 2012], [Sawada et al., 2013], [Higuchi and Kameoka, 2014]

ILRMA: Independent Low-Rank Matrix Analysis

- Earlier idea (determined multi-channel NMF) [Kameoka et al., 2010]
- Multichannel NMF with rank-1 spatial model [Kitamura et al., 2015a]
- ILRMA [Kitamura et al., 2016], [Kitamura et al., 2018]
- Relaxation of Rank-1 spatial model [Kitamura et al., 2015b]
- Maximization-equalization algorithm [Mitsui et al., 2017b]
- Optimal window length [Kitamura et al., 2017]
- Based on Student's t-distribution [Mogami et al., 2017]
- With sparse regularization [Mitsui et al., 2017a]
- With spatial regularization [Mitsui et al., 2018]

Other references related to auxiliary function based optimization

- [Lange et al., 2000], [Hunter and Lange, 2000], [Hunter and Lange, 2004], [Ono et al., 2009], [Ono and Sagayama, 2010], [Févotte and Idier, 2011], [Yoshii et al., 2013], [Kameoka and Takamune, 2014], [Sun et al., 2017]

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